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Name of the Student: Branch:

Unit – I (Random Variables)

• Problems on Discrete & Continuous R.Vs

1. A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(1) Find the value of k .

(2) Evaluate $p(X<6), p(X \geq 6)$

(3) If $p(X \leq c) > \frac{1}{2}$ find the minimum value of c . (M/J 2012)

Textbook Page No.: 1.8

2. A random variable X has the following probability distribution

X	–2	–1	0	1	2	3
P(x)	0.1	K	0.2	2k	0.3	3k

(1) Find k , (2) Evaluate $p(X<2)$ and $p(-2<X<2)$, (3) Find the PDF of X and (4) Evaluate the mean of X . (N/D 2011)

Textbook Page No.: 1.10

3. A random variable X takes the values $-2, -1, 0$ and 1 with probabilities $\frac{1}{8}, \frac{1}{8}, \frac{1}{4}$ and $\frac{1}{2}$ respectively. Find the mean and variance. (N/D 2014)

Textbook Page No.: 1.11

4. If the random variable X takes the values 1,2,3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$, then find the probability distribution and cumulative distribution function of X. (N/D 2012)

Textbook Page No.: 1.12

5. The probability function of an infinite discrete distribution is given by $p(X=j)=\frac{1}{2^j};\ j=1,2...,\infty$. Verify that the total probability is 1 and find the mean and variance of the distribution. Find also $p(X \text{ is even}), p(X \geq 5)$ and $p(X \text{ is divisible by } 3)$. (N/D 2011)

Textbook Page No.: 1.13

6. Let X be a continuous random variable with the probability density function $f(x)=\frac{1}{4},\ 2 \leq x \leq 6$. Find the expected value and variance of X. (N/D 2017)

Textbook Page No.: 1.14

7. If $f(x)=\begin{cases} xe^{-x^2/2}; & x \geq 0 \\ 0 & ; x<0 \end{cases}$, then show that $f(x)$ is a pdf and find $F(x)$. (N/D 2014)

Textbook Page No.: 1.17

8. The distribution function of a random variable X is given by $F(X)=1-(1+x)e^{-x};\ x \geq 0$. Find the density function, mean and variance of X.

Textbook Page No.: 1.19 (N/D 2010)

9. If X is a random variable with a continuous distribution function $F(X)$, prove that $Y=F(X)$ has a uniform distribution in $(0,1)$. Further if

$$f(X)=\begin{cases} \frac{1}{2}(x-1); & 1 \leq x \leq 3 \\ 0; & \text{otherwise} \end{cases},$$

find the range of Y corresponding to the range $1.1 \le x \le 2.9$. (N/D 2010)

10. If the density function of X equals $f(x)=\begin{cases} Ce^{-2x}, & 0<x<\infty \\ 0, & x<0 \end{cases}$, find C. What is $P[X>2]$? (A/M 2010)

Textbook Page No.: 1.20

11. A continuous random variable has the pdf $f(x)=kx^4, \quad -1<x<0$. Find the value of k and also $p\left\{X>\left(\frac{-1}{2}\right)/X<\left(\frac{-1}{4}\right)\right\}$. (M/J 2013)

Textbook Page No.: 1.21

12. The DF of a continuous random variable X is given by

$$F(x)=\begin{cases} 0, & x<0 \\ x^2, & 0\le x\le \frac{1}{2} \\ 1-\frac{3}{25}\left(3-x^2\right), & \frac{1}{2}\le x\le 3 \\ 1, & x\ge 3 \end{cases}$$

. Find the pdf of X and evaluate $p\left(|X|\le 1\right)$ and $p\left(\frac{1}{3}<X<4\right)$ using both the CDF and PDF. (N/D 2011)

Textbook Page No.: 1.24

• Moments and Moment Generating Functions

1. Find the MGF of the binomial distribution and hence find its mean. (N/D 2012)

Textbook Page No.: 1.33

2. By calculating the moment generation function of Poisson distribution with parameter λ, prove that the mean and variance of the Poisson distribution are equal. (A/M 2010), (N/D 2015)

3. Find the MGF of a Poisson random variable and hence find its mean and variance. (N/D 2014)

4. Determine the mean, variance and moment generating function of a random variable X following Poisson distribution with parameter λ. (M/J 2014)

Textbook Page No.: 1.34

5. Describe the situations in which geometric distributions could be used. Obtain its moment generating function. (A/M 2010)

Textbook Page No.: 1.36

6. Derive mean and variance of a Geometric distribution. Also establish the forgetfulness property of the Geometric distribution. (A/M 2011)

Textbook Page No.: 1.36

7. Find the moment generating function of Uniform distribution Hence find its mean and variance. (M/J 2013)

Textbook Page No.: 1.37

8. Define Gamma distribution and find the mgf, mean and variance.(N/D 2011),(N/D 2017)

9. Find the moment generating function of an exponential random variable and hence find its mean and variance. (M/J 2012),(M/J 2014)

Textbook Page No.: 1.40

10. Find the moment generating function of a normal distribution and hence find its mean and variance. (N/D 2017)

Textbook Page No.: 1.41

11. Find the moment generating function and rth moment for the distribution whose pdf is $f(x) = ke^{-x},\ \ 0 \le x \le \infty$. Hence find the mean and variance. (M/J 2013)

Textbook Page No.: 1.44

12. A continuous random variable X has the Pdf $f(x) = kx^3 e^{-x}$, $x > 0$. Find the r^{th} order moment about the origin, moment generating function, mean and variance of X.

Textbook Page No.: 1.45 (N/D 2015)

13. Find the MGF of the random variable 'X' having the pdf $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$.

Textbook Page No.: 1.47 (N/D 2013)

• Problems on Distributions

1. The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without a breakdown (2) with only one breakdown.(N/D 2012)

   Textbook Page No.: 1.50

2. A manufacture of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective, what is the probability that a box fail to meet the guaranteed quality? (N/D 2013)

   Textbook Page No.: 1.51

3. In a large consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) at most there are 3 defective bulbs (3) exactly there are 3 defective bulbs. (M/J 2013)

   Textbook Page No.: 1.52

4. Messages arrive at switchboard in a Poisson manner at an average rate of six per hour. Find the probability that atleast three messages arrive within one hour. (N/D 2017)

5. If X is a Poisson variate such that $p(X=2)=9p(X=4)+90p(X=6)$. Find

   (1) Mean and $E(X^2)$ (2) $p(X \geq 2)$ (M/J 2012)

   Textbook Page No.: 1.53

6. Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7, (M/J 2014)

   (i) What is the probability that the target would be hit on tenth attempt?
   (ii) What is the probability that it takes him less than 4 shots?
   (iii) What is the probability that it takes him an even number of shots?

   Textbook Page No.: 1.57

7. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (1) on the 4^{th} trial (2) in fewer than 4 trials? (N/D 2012)

   Textbook Page No.: 1.58

8. A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p'. Find the value of 'p' so that the probability

that an odd number of tosses required is equal to 0.6. Can you find a value of 'p' so that the probability is 0.5 that an odd number of tosses are required? (N/D 2010)

Textbook Page No.: 1.59

9. Trains arrive at a station at 15 minutes intervals starting at 4 a.m. If a passenger arrive at a station at a time that is uniformly distributed between 9.00 and 9.30, find the probability that he has to wait for the train for (1) less than 6 minutes (2) more than 10 minutes. (M/J 2014)

Textbook Page No.: 1.60

10. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$ .What is the probability that the repair time exceeds 2h? What is the conditional probability that a repair takes at least 10h given that its duration exceeds 9h? (N/D 2010)

Textbook Page No.: 1.64

11. In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a random variable having Gamma distribution with parameters $\lambda = \frac{1}{2}$ and $v = 3$. If the power plant of this city has a daily capacity of 12 millions kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? (M/J 2012)

12. If a continuous RV, X follows uniform distribution in the interval $(0, 2)$ and a continuous RV, Y follows exponential distribution with parameter λ. Find λ such that $P(X < 1) = P(Y < 1)$. (N/D 2013)

Textbook Page No.: 1.66

13. A component has an exponential time to failure distribution with mean of 10,000 hours.

(1) The component has already been in operation for its mean life. What is the probability that it will fail by 15,000 hours?

(2) At 15,000 hours the component is still in operation. What is the probability that it will operate for another 5000 hours? (N/D 2015)

Textbook Page No.: 1.68

14. The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that two of them will have marks over 70? (A/M 2010),(A/M 2011)

Textbook Page No.: 1.70

15. Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable X normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by

    (1) at least 44.5 cc/min

    (2) at most 35.0 cc/min

    (3) anywhere from 30.0 to 40.0 cc/mm. (N/D 2012)

Textbook Page No.: 1.71

16. The average percentage of marks of candidates in an examination in 42 with a standard deviation of 10. If the minimum mark for pass is 50% and 1000 candidates appear for the examination, how many candidates can be expected to get the pass mark if the marks follow normal distribution? If it is required, that double the number of the candidates should pass, what should be the minimum mark for pass? (N/D 2015)

17. Let X and Y be independent normal variates with mean 45 and 44 and standard deviation 2 and 1.5 respectively. What is the probability that randomly chosen values of X and Y differ by 1.5 or more? (N/D 2011)

Textbook Page No.: 1.73

18. Given that X is distributed normally, if $P(X < 45) = 0.31$ and $P(X > 64) = 0.08$, find the mean and standard deviation of the distribution. (M/J 2012)

Textbook Page No.: 1.76

19. If X and Y are independent random variables following $N(8,2)$ and $N\left(12,4\sqrt{3}\right)$ respectively, find the value of λ such that $P\left[2X - Y \le 2\lambda\right] = P\left[X + 2Y \ge \lambda\right]$.

Textbook Page No.: 1.78 (N/D 2010)

Unit – II (Two Dimensional Random Variables)

• Joint distributions – Marginal & Conditional

1. The joint distribution of X and Y is given by $f(x,y)=\frac{x+y}{21}$, $x=1,2,3$, $y=1,2$. Find the marginal distribution and conditional distributions. (N/D 2013),(N/D 2015)

   Textbook Page No.: 2.1

2. The joint probability mass function of (X,Y) is given by $p(x,y)=K(2x+3y)$, $x=0,1,2;\ y=1,2,3$. Find all the marginal and conditional probability distributions.

   Textbook Page No.: 2.3 (N/D 2011)

3. Let X and Y be two random variables having the joint probability function $f(x,y)=k(x+2y)$ where x and y can assume only the integer values 0, 1 and 2. Find the marginal and conditional distributions. (M/J 2012)

   Textbook Page No.: 2.5

4. The joint probability density function of two random variables X and Y is given by $f(x,y)=\begin{cases}\frac{6}{7}\left(x^2+\frac{xy}{2}\right), & 0\le x\le 1,\ 0\le y\le 2\\ 0, & \text{otherwise}\end{cases}$. Find the conditional density function of X given Y and the conditional density function of Y given X. (M/J 2014)

   Textbook Page No.: 2.6

5. Given $f(x,y)=cx(x-y),\ 0<x<2,\ -x<y<x$ and '0' elsewhere. Evaluate 'c' and find $f_X(x)$ and $f_Y(y)$ respectively. Compute $P[X<Y]$. (N/D 2010)

   Textbook Page No.: 2.8

6. The joint density of X and Y is given by $f(x,y)=\begin{cases}\frac{1}{2}ye^{-xy}, & x>0, 0<y<2\\ 0, & \text{otherwise}\end{cases}$. Calculate the conditional density of X given $Y=1$. (A/M 2010)

   Textbook Page No.: 2.12

7. The joint pdf of random variable X and Y is given by $f(x,y)=\begin{cases}\lambda xy^2, & 0\le x\le y\le 1\\ 0, & \text{otherwise}\end{cases}$. (1) Determine the value of λ. (2) Find the marginal probability density function of X. (N/D 2012)

Textbook Page No.: 2.13

8. Given the joint density function $f(x,y)=\begin{cases}x\dfrac{(1+3y^2)}{4}, & 0<x<2, 0<y<1\\ 0, & \text{elsewhere}\end{cases}$. Find the marginal densities $g(x), h(y)$ and the conditional density $f(x/y)$ and evaluate $P\left[\frac{1}{4}<x<\frac{1}{2}/Y=\frac{1}{3}\right]$. (A/M 2011)

Textbook Page No.: 2.15

9. Determine whether the random variables X and Y are independent, given their joint probability density function as $f(x,y)=\begin{cases}x^2+\dfrac{xy}{3}, & 0\le x\le 1,\ 0\le y\le 2\\ 0, & \text{otherwise}\end{cases}$.

Textbook Page No.: 2.16 (A/M 2011)

10. The joint probability density function of a two-dimensional random variable (X,Y) is $f(x,y)=\frac{1}{8}(6-x-y),\ \ 0<x<2,\ 2<y<4$. Find (1) $P(X<1\cap Y<3)$ (2) $P(X+Y<3)$ (3) $P(X<1/Y<3)$ (M/J 2013)

Textbook Page No.: 2.18

11. Two dimensional random variables (X,Y) have the joint probability density function

$$f(x,y)=8xy,\ \ 0<x<y<1$$
$$=0, \text{elsewhere}.$$

(1) Find $P\left(X<\frac{1}{2}\cap Y<\frac{1}{4}\right)$.

(2) Find the marginal and conditional distributions.

(3) Are X and Y independent? (M/J 2012)

Textbook Page No.: 2.21

12. Two random variables X and Y have the following joint probability density function $f(x,y) = xe^{-x(y+1)},\ x \le 0,\ y \ge 0$. Determine the conditional probability density function of X given Y and the conditional probability density function of Y given X.

Textbook Page No.: 2.123 (N/D 2017)

• Covariance, Correlation and Regression

1. Obtain the equations of the lines of regression from the following data: (N/D 2012)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

Textbook Page No.: 2.29

2. Find the correlation coefficient for the following data: (N/D 2011)

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

Textbook Page No.: 2.26

3. The marks obtained by 10 students in Mathematics and Statistics are given below. Find the correlation coefficient between the two subjects. (M/J 2013)

Marks in Maths	75	30	60	80	53	35	15	40	38	48
Marks in Stats.	85	45	54	91	58	63	35	43	45	44

Textbook Page No.: 2.27

4. Calculate the coefficient of correlation for the following data: (N/D 2014)

X:	9	8	7	6	5	4	3	2	1
Y:	15	16	14	13	11	12	10	8	9

Textbook Page No.: 2.28

5. Compute the coefficient of correlation between X and Y using the following data: (N/D 2010)

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

Textbook Page No.: 2.28

6. Two random variables X and Y have the joint probability density function $f(x,y)=\begin{cases} c(4-x-y), & 0\le x\le 2,\ 0\le y\le 2 \\ 0, & \text{elsewhere} \end{cases}$. Find $\text{cov}(X,Y)$ and the equations of two lines of regression. (M/J 2012),(N/D 2016)

Textbook Page No.: 2.30

7. Let X and Y be random variables having joint density function $f(x,y)=\begin{cases} \frac{3}{2}\left(x^2+y^2\right), & 0\le x\le 1,\ 0\le y\le 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the correlation co-efficient r_{xy}.

Textbook Page No.: 2.34 (N/D 2013)

8. The joint probability density function of two random variables X and Y is given by $f(x,y)=\begin{cases} k\left[\left(x+y\right)-\left(x^2+y^2\right)\right], & 0<(x,y)<1 \\ 0, & \text{otherwise} \end{cases}$. Show that X and Y are uncorrelated but not independent. (M/J 2014)

Textbook Page No.: 2.37

9. The regression equation of X and Y is $3y-5x+108=0$. If the mean value of Y is 44 and the variance of X were 9/16 th of the variance of Y. Find the mean value of X and the correlation coefficient. (N/D 2012)

Textbook Page No.: 2.41

10. If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient, r_{UV} where $U=X+Y$ and $V=X-Y$.

Textbook Page No.: 2.42 (M/J 2014)

• Transformation of random variables

1. If the pdf of 'X' is $f_X(x)=2x,\ 0<x<1$, find the pdf of $Y=3X+1$. (N/D 2013)

Textbook Page No.: 2.45

2. If X and Y are independent RVs with pdf's $e^{-x}, x\ge 0,$ and $e^{-y}, y\ge 0,$ respectively, find the density functions of $U=\dfrac{X}{X+Y}$ and $V=X+Y$. Are U and V independent?

Textbook Page No.: 2.46 (N/D 2011),(N/D 2015)

3. If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (M/J 2013)

Textbook Page No.: 2.48

4. Let X and Y be independent random variables both uniformly distributed on $(0,1)$. Calculate the probability density of $X + Y$. (A/M 2010)

Textbook Page No.: 2.51

5. If X and Y are independent random variables having density functions $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ and $f(y) = \begin{cases} 3e^{-3y}, & y \geq 0 \\ 0, & y < 0 \end{cases}$, respectively, find the density functions of $z = X - Y$. (A/M 2011)

Textbook Page No.: 2.52

Unit – III (Random Processes)

- **Verification of SSS and WSS process**

1. Show that the random process $X(t) = A\cos(\omega t + \theta)$ is a Wide Sense Stationary Process if A and ω are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (N/D 2011),(M/J 2014),(N/D 2015)

Textbook Page No.: 3.2

2. Show that the process $X(t) = A\cos\lambda t + B\sin\lambda t$ is wide sense stationary, if $E(A) = E(B) = 0,\ E(A^2) = E(B^2)$ and $E(AB) = 0$, where A and B are random variables. (M/J 2013),(N/D 2018)

Textbook Page No.: 3.4

3. Show that random process $\{X(t)\} = A\cos t + B\sin t,\ -\infty < t < \infty$ is a wide sense stationary process where A and B are independent random variables each of which has a value -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. (A/M 2011)

Textbook Page No.: 3.5

4. The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P[X(t)=n]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,...\\ \dfrac{at}{1+at}, & n=0\end{cases}$$

. Show that $\{X(t)\}$ is not stationary.

Textbook Page No.: 3.9 (M/J 2012),(N/D 2013),(N/D 2015),(N/D 2017)

• Problems on Markov Chain

1. An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follow a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly signals are not recognizable, find the fraction of signals that are highly distorted.

   Textbook Page No.: 3.11 (N/D 2010),(N/D 2014)

2. An observer at a lake notices that when fish are caught, only 1 out of 9 trout is caught after another trout, with no other fish between, whereas 10 out of 11 non-trout are caught following non-trout, with no trout between. Assuming that all fish are equally likely to be caught, what fraction of fish in the lake is trout? (N/D 2012)

   Textbook Page No.: 3.12

3. A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. (N/D 2011),(N/D 2015)

   Textbook Page No.: 3.13

4. A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city-A, then the next day he sells in city-B. However if he sells in either city-B or city-C, the next day he is twice as likely to sell in city-A as in the other city. In the long run how often does he sell in each of the cities?

   Textbook Page No.: 3.15 (M/J 2012),(N/D 2013)

5. A soft water plant works properly most of the time. After a day in which the plant is working, the plant is working the next day with probability 0.95. Otherwise a day or repair followed by a day of testing is required to restore the plant to working status. Draw the state transition diagram for the status of the plant. Write down the tpm and classify the status of the process. (N/D 2014)

6. A fair die is tossed repeatedly. The maximum of the first 'n' outcomes is denoted by X_n. Is $\{X_n : n = 1, 2, ...\}$ a Markov chain? Why or why not? If it is a Markov chain, calculate its transition probability matrix. Specify the classes. (N/D 2012)

7. Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Show how this system may be analyzed by using a Markov chain. How many states are needed? (A/M 2010)

8. Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks? (A/M 2010)

9. Let the Markov Chain consisting of the states 0, 1, 2, 3 have the transition probability matrix $P = \begin{pmatrix} 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$. Determine which states are transient and which are recurrent by defining transient and recurrent states. (A/M 2010)

Textbook Page No.: 3.23

10. Find the limiting-state probabilities associated with the following transition probability matrix. $\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$. (A/M 2011)

Textbook Page No.: 3.17

11. Consider a Markov chain with transition probability matrix $P = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$. Find the limiting probabilities of the system. (M/J 2014)

Textbook Page No.: 3.18

12. The transition probability matrix of a Markov chain $\{X(t)\},\ n = 1,2,3,...$, having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $p^{(0)} = (0.7 \quad 0.2 \quad 0.1)$. Find (1) $p[X_2 = 3]$ (2) $p[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (M/J 2012),(N/D 2013),(M/J 2014)

Textbook Page No.: 3.19

13. The following is the transition probability matrix of a Markov chain with state space $\{1,2,3,4,5\}$. Specify the classes, and determine which classes are transient and which are recurrent. $P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1/3 & 0 & 2/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2/5 & 0 & 3/5 \end{pmatrix}$ (N/D 2012)

14. Find the nature of the states of the Markov chain with the tpm $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$.

Textbook Page No.: 3.22 (M/J 2013)

15. Consider a Markov chain with 3 states and transition probability matrix $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$. Find the stationary probabilities of the chain. (N/D 2017)

16. The following is the transition probability matrix of a Markov chain with state space $\{0,1,2,3,4\}$. Specify the classes, and determine which classes are transient and which are recurrent. Give reasons. $P = \begin{pmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{pmatrix}$ (N/D 2010)

Textbook Page No.: 3.20

17. A gambler has Rs.2. He bets Rs.1 at a time and wins Rs.1 with probability $1/2$. He stops playing if he loses Rs.2 or wins Rs.4. (1) What is the tpm of the related Markov chain? (2) What is the probability that he has lost his money at the end of 5 plays?(M/J 2013)

Textbook Page No.: 3.23

- **Poisson process**

1. Define Poisson process and derive the Poisson probability law. (N/D 2011)

   Textbook Page No.: 3.26

2. Derive probability distribution of Poisson process and hence find its auto correlation function. (A/M 2011)

   Textbook Page No.: 3.26

3. Show that the difference of two independent Poisson processes is not a Poisson process. (A/M 2011),(M/J 2013)

   Textbook Page No.: 3.31

4. If $\{X_1(t)\}$ and $\{X_2(t)\}$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, show that the process $\{X_1(t)+X_2(t)\}$ is also a Poisson process.

   Textbook Page No.: 3.30 (M/J 2014)

5. Prove that the Poisson process is a Markov process. (M/J 2013)

   Textbook Page No.: 3.32

6. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2/min, find the probability that the interval between 2 consecutive arrivals is more than 1 min, between 1 and 2 mins, and 4 mins or less. (N/D 2010),(N/D 2011)

   Textbook Page No.: 3.34

7. Suppose that children are born at a Poisson rate of five per day in a certain hospital. What is the probability that (1) atleast two babies are born during the next six hours, (2) no babies are born during the next two days? (N/D 2014)

8. Suppose that telephone calls arriving at a particular switchboard follow a Poisson process with an average of 5 calls coming per minute. What is the probability that up to a minute will elapse unit 2 calls have come in to the switch board? (A/M 2011)

9. Suppose the arrival of calls at a switch board is modeled as a Poisson process with the rate of calls per minute being $\lambda = 0.1$. What is the probability that the number of calls arriving in a 10 minutes interval is less than 3? (N/D 2017)

Textbook Page No.: 3.36

10. Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes

(1) Exactly 4 customers arrive (2) More than 4 customers arrive

(3) Less than 4 customers arrive. (M/J 2012),(N/D 2013),(N/D 2015)

Textbook Page No.: 3.36

Unit – IV (Queueing Models)

- **Model – I (M/M/1) : (∞/FIFO)**

1. Customers arrive at a sale counter managed by a single person according to a Poisson process with mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer. (N/D 2013)

Textbook Page No.: 4.14

2. Customer arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then (M/J 2013)

(i) What is the probability that a customer need not wait for a haircut?
(ii) What is the expected number of customer in the barber shop and in the queue?
(iii) How much time can a customer expect to spend in the barber shop?
(iv) Find the average time that a customer spend in the queue.
(v) Estimate the fraction of the day that the customer will be idle?
(vi) What is the probability that there will be 6 or more customers?
(vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair.

Textbook Page No.: 4.15

3. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it takes exactly 1.5 min to reach the correct seat after purchasing the ticket, (N/D 2010),(N/D 2014)

   (i) Can he expect to be seated for the start of the picture?

   (ii) What is the probability that he will be seated for the start of the picture?

   (iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture?

4. A T.V. repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day. Find (M/J 2012),(N/D 2013),(N/D 2015)

   (1) the repairman's expected idle time each day

   (2) how many jobs are ahead of average set just brought?

Textbook Page No.: 4.19

5. Customers arrive at the express checkout lane in a supermarket in a Poisson process with a rate of 15 per hour. The time to check out a customer is an exponential random variable with mean of 2 minutes. Find the average number of customers present. What is the expected waiting time for a customer in the system? (M/J 2014)

Textbook Page No.: 4.20

- ## Model – II (M/M/s) : (∞/FIFO)

1. There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,

   (i) What fraction of the time all the typists will be busy?

   (ii) What is the average number of letters waiting to be typed?

   (iii) What is the average time a letter has to spend for waiting and for being typed? (N/D 2010),(N/D 2011)

Textbook Page No.: 4.23

2. A supermarket has 2 girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, find the following: (M/J 2012)

   (1) What is the probability of having to wait for service?

   (2) What is the expected percentage of idle time for each girl?

   (3) What is the expected length of customer's waiting time?

Textbook Page No.: 4.25

3. Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 5 trains, find the probability that the yard is empty and the average number of trains in the system, given that the inter arrival time and service time are following exponential distribution. (M/J 2012)

4. There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour, what fraction of times all the typists will be busy? What is the average number of letters waiting to be typed? (M/J 2012),(M/J 2013)

5. Four counters are being run on the frontiers of the country to check the passports of the tourists. The tourists choose a counter at random. If the arrival at the frontier is Poisson at the rate λ and the service time is exponential with parameter $\frac{\lambda}{2}$, find the average queue length at each counter. (M/J 2014)

Textbook Page No.: 4.27

- ## Model – III (M/M/1) : (K/FIFO)

1. The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4/hr (exponential service time) (N/D 2014)

   (i) What percentage of time is the barber idle?

   (ii) What fraction of the potential customers are turned away?

   (iii) What is the expected number of customers waiting for a hair-cut?

   (iv) How much time can a customer expect to spend in the barber shop?

Textbook Page No.: 4.31

2. Patients arrive at a clinic according to Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.
(1) Find the effective arrival rate at the clinic (2) What is the probability that an arriving patient will not wait? (3) What is the expected waiting time until a patient is discharged from the clinic? (N/D 2015)(A/M 2018)

Textbook Page No.: 4.33

3. Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space is front of window, including that for the serviced car can accommodate a maximum of three cars. Others cars can wait outside this space. (A/M 2011)

    (1) What is the probability that an arriving customer can drive directly to the space in front of the window?

    (2) What is the probability that an arriving customer will have to wait outside the indicated space?

    (3) How long is an arriving customer expected to wait before being served?

Textbook Page No.: 4.36

4. Consider a single server queueing system with Poisson input, exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hours and the maximum permissible number calling units in the system is two. Find the steady state probability distribution of the number of calling units in the system and the expected number of calling units in the system. (N/D 2013),(N/D 2017)

Textbook Page No.: 4.38

5. A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour. What is the probability that an arrival would have to wait in line? Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. (N/D 2018)

- **Model – IV (M/M/c) : (K/FIFO)**

1. The engineers have two terminals available to aid their calculations. The average computing job requires 20 minutes of terminal time and each engineer requires some computation one in half an hour. Assume that these are distributed according to an exponential distribution. If the terminals can accommodate only 6 engineers in the waiting space find the expected number of engineers in the computing center. (N/D 2015)

Textbook Page No.: 4.43

2. At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average, 10 hrs to unload a tanker, the unloading time following an exponential distribution. Find (N/D 2012)
   (i) how many tankers are at the port on the average?
   (ii) how long does a tanker spend at the port on the average?
   (iii) what is the average arrival rate at the overflow facility?

   Textbook Page No.: 4.48

3. Let there be an automobile inspection situation with three inspection stalls. Assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate almost four cars waiting at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with a mean of 6 minutes. Find the average number of customers in the system during the peak hours, the average waiting time and the average number per hours that cannot enter the station because of full capacity. (N/D 2017)

• Derivations in Queueing Models

1. Define birth and death process. Obtain its steady state probabilities. How it could be used to find the steady state solution for the M/M/1 model? Why is it called geometric? (A/M 2010)

   Textbook Page No.: 4.1

2. Obtain the steady state probabilities of birth-death process. Also draw the transition graph. (N/D 2012)

   Textbook Page No.: 4.1

3. Derive the governing equations for the $(M/M/1):(GD/N/\infty)$ queueing model and hence obtain the expression for the steady state probabilities and the average number of customers in the system. (M/J 2014)

   Textbook Page No.: 4.4

4. Derive the steady-state probabilities of the number of customers in M/M/1 queueing system from the birth and death processes and hence deduce that the average measures such as expected system size L_s, expected queue size L_q, expected waiting time in system W_s and expected waiting time in queue W_q. (N/D 2018)

   Textbook Page No.: 4.4

5. Show that for a single service station, Poisson arrivals and exponential service time, the probability that exactly n calling units in the queueing system is $P_n = (1-\rho)\rho^n,\ n \geq 0$, where ρ is the traffic intensity. Also, find the expected number of units in the system.

   Textbook Page No.: 4.4 (N/D 2017)

6. Derive the governing equations for the $(M/M/C):(\mathbf{GD}/\infty/\infty)$ queueing model and hence obtain the expression for the steady state probabilities and the average number of customers in the system. (M/J 2014)

   Textbook Page No.: 4.7

7. Find the system size probabilities for an $(M/M/C):(FIFO/\infty/\infty)$ queueing system under steady state conditions. Also obtain the expression for average number of customers in the system. (N/D 2015)

   Textbook Page No.: 4.7

8. Derive (1) L_s, average number of customers in the system (2) L_q, average number of customers in the queue for the queuing model (M/M/1):(N/FIFO). (M/J 2013)

9. Calculate any four measures of effectiveness of M/M/1 queueing model. (A/M 2010)

   Textbook Page No.: 4.4

10. Show that for the $(M/M/1):(FCFS/\infty/\infty)$, the distribution of waiting time in the system is $w(t) = (\mu-\lambda)e^{-(\mu-\lambda)t},\ t > 0$. (A/M 2011)

    Textbook Page No.: 4.7

11. Find the steady state solution for the multiserver M/M/C model and hence find L_q, W_q, W_s and L_s by using Little formula. (A/M 2011), (A/M 2018)

    Textbook Page No.: 4.8

Unit – V (Advanced Queueing Models)

• Pollaczek – Khinchine formula

1. Derive Pollaczek – Khintchine formula of M/G/1 queue. (A/M 2010),(N/D 2010),(N/D 2011),(M/J 2012),(N/D 2012),(N/D 2013),(N/D 2014),(N/D 2017),(N/D 2018)

   Textbook Page No.: 5.1

2. Derive the expected steady state system size for the single server queues with Poisson input and General Service. (A/M 2011)

   Textbook Page No.: 5.1

3. A one man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. (N/D 2013),(N/D 2015)

   Textbook Page No.: 5.6

4. Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes, determine (M/J 2012),(M/J 2013)

   (1) mean number of customers in the system L_s (M/J 2014)

   (2) mean number of customers in the queue L_q

   (3) mean waiting time of a customer in the system W_s

   (4) mean waiting time of a customer in the queue W_q

   Textbook Page No.: 5.7

5. A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for a car has uniform distribution between 8 and 12 minutes , Find (N/D 2013)

   (1) The average number of cars waiting in the parking lot (M/J 2014)

(2) The average waiting time of a car in the parking lot.

Textbook Page No.: 5.8

6. In a factory cafeteria, the customers have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at the third. The server at each counter takes, on an average, 1.5 minutes, although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Calculate (i) the average time a customer spend waiting in the cafeteria (ii) the average time of getting the service (iii) the most probable time in getting the service. (N/D 2017)

Textbook Page No.: 5.16

- **Queueing networks**

1. Consider a two stage tandem queue with external arrival rate λ to node '0'. Let μ_0 and μ_1 be the service rates of the exponential servers at node '0' and '1' respectively. Arrival process is Poisson. Model this system using a Markov chain and obtain the balance equations. (N/D 2012)

2. Write short notes on the following :

   (i) Queue networks

   (ii) Series queues

   (iii) Open networks (N/D 2014)

   (iv) Closed networks (N/D 2010),(A/M 2011)

7. Discuss open and closed networks. (N/D 2011)

- **Series Queue**

8. There are two salesmen in a ration shop one incharge of billing and receiving payment and the other incharge of weighing and delivering the items. Due to limited availability of space, only one customer is allowed to enter the shop, that too when the billing clerk is free. The customer who has finished his billing job has to wait there until the delivery section becomes free. If customers arrive in accordance with a Poisson process at rate 1 and the service times of two clerks are independent and have exponential rate of 3 and 2 find (N/D 2013)

   (1) The proportion of customers who enter the ration shop.
   (2) The average number of customers in the shop.

(3) The average amount of time that an entering customer spends in the shop.

Textbook Page No.: 5.19

9. An average of 120 students arrive each hour (inter-arrival times are exponential) at the controller office to get their hall tickets. To complete the process, a candidate must pass through three counters. Each counter consists of a single server, service times at each counter are exponential with the following mean times: counter 1, 20 seconds; counter 2, 15 seconds and counter 3, 12 seconds. On the average how many students will be present in the controller's office. (M/J 2012),(M/J 2014)

Textbook Page No.: 5.24

10. In a big factory, there are a large number of operating machines and sequential repair shops which do the service of the damaged machines exponentially with respective 1/hour and 2/hour. If the cumulative failure rate of all the machines in the factory is 0.5/hour, find (1) the probability that both repair shops are idle (2) the average number of machines in the service. (N/D 2017)

Textbook Page No.: 5.23

- **Open and Closed Network**

11. Consider two servers. An average of 8 customers per hour arrive from outside at server 1 and an average of 17 customers per hour arrive from outside at server 2. Inter arrival times are exponential. Server 1 can serve at an exponential rate of 20 customers per hour and server 2 can serve at an exponential rate of 30 customers per hour. After completing service at station 1, half the customers leave the system and half go to server 2. After completing service at station 2, 3/4 of the customer complete service and 1/4 return to server 1. Find the expected no. of customers at each server. Find the average time a customer spends in the system. (N/D2012)

Textbook Page No.: 5.26

12. Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. (M/J 2013),(N/D 2015),(N/D 2018)

Textbook Page No.: 5.28

13. For an open queueing network with three nodes 1, 2 and 3, let customers arrive from outside the system to node j according to a Poisson input process with parameters r_j and let P_{ij} denote the proportion of customers departing from facility i to facility j. Given $(r_1, \quad r_2, \quad r_3) = (1, \quad 4, \quad 3)$ and $P_{ij} = \begin{bmatrix} 0 & 0.6 & 0.3 \\ 0.1 & 0 & 0.3 \\ 0.4 & 0.4 & 0 \end{bmatrix}$ determine the average arrival rate λ_j to the node j for $j = 1, 2, 3$. (M/J 2012)

Textbook Page No.: 5.31

Book for Reference:

"Probability and Queueing Theory"

Edition: 1st Edition

Publication: Sri Hariganesh Publications **Author: C. Ganesan**

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