B.Arch. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2011

Common to: B.Arch. and B.Arch. (Interior Design)

First Semester

381101 – MATHEMATICS

(Regulation 2010)

Time: Three hours Maximum: 100 marks

Answer ALL questions

PART A – (10 X 2 = 20 marks)

1. Find the characteristic equation of $A = \begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$.
2. Convert the quadratic form $4x^2 + 5y^2 - 7xy + 4yz$ to matrix form.
3. Find the equation of the tangent plane at $(1,1,-2)$ to the sphere $x^2 + y^2 + z^2 - 2x - y - z - 5 = 0$.
4. Find the angle between the planes $2x + 3y - 5z = 6,\ 3x + 8y + 6z = 9$.
5. Find the radius of curvature of the curve $y = e^x$ at $(0,1)$.
6. Obtain the envelope of the family of the curves $y = mx + \frac{a}{m}$.
7. If $u = x^2 + y^2 + z^2$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u$.
8. Find the stationary points of $f(x, y) = x^2 + y^2 + 6x + 12$.
9. Solve $\frac{d^2y}{dx^2} + 9y = \cosh x$.
10. Transform the equation $x^2y'' - xy' + y = 0$ into a linear equation with constant coefficients.

PART B – (5 X 16 = 80 marks)

11. (a) (i) Show that the matrix $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$ satisfies the equation $A(A-I)(A+2I) = 0$.

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(ii) Using Cayley-Hamilton theorem, find the inverse of $A=\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$.

Or

(b) (i) Diagonalise the matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ by means of orthogonal transformation.

(ii) Find the Eigen values and Eigen vectors of $A=\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$.

12. (a) (i) Find the equation of the circular plane which passes through the line of intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and perpendicular to the plane $2y-3z=4$.

(ii) Show that the spheres $x^2+y^2+z^2+6y+2z+8=0$, $x^2+y^2+z^2+6x+8y+4z+20=0$ intersect at right angles. Find the plane of intersection.

Or

(b) (i) Find the equation of the plane through the points $(1,0,-1)$ and $(3,2,2)$ and parallel to the line $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

(ii) Find the equation of the sphere through the circle $x^2+y^2+z^2-2x+3y+4z-5=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$ and passing through the point $(1,1,2)$.

13. (a) (i) Find the radius of curvature of $\frac{x^2}{9}+\frac{y^2}{4}=1$ at $(3,0)$.

(ii) Find the equation of the evolute of the parabola $y^2=4x$.

Or

(b) (i) Find the envelope of the family of curves $\frac{x^3}{a^3}\sec\theta+\frac{y^3}{b^3}\cos ec\theta=1, \theta$ being the parameter.

(ii) Show that the radius of curvature of $x=e^t\cos t,\ y=e^t\sin t$ is $\sqrt{2}t$.

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14. (a) (i) If $u = \frac{y^2}{2x}, \ v = \frac{x^2 + y^2}{2x}$, show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{-y}{2x}$.

(ii) Use Taylor's series theorem to expand $f(x,y) = x^2 + xy + y^2$ in powers of $(x-1)$ and $(y-2)$.

Or

(b) (i) The temperature T at a point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

(ii) If $u = f(r), \ x = r\cos\theta, \ y = r\sin\theta$, show that $u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r)$.

15. (a) (i) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^3 + \cos 2x$.

(ii) Solve the system of equations $\frac{dy}{dt} + 2x + y = 0, \ \frac{dx}{dt} + 5x - 2y = t$ given that $x = y = 0$ when $t = 0$.

Or

(b) (i) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x\sin x$.

(ii) Solve $\left(x^2D^2 - xD - 3\right)y = \frac{1}{x}\cos\left(2\log x\right)$.

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