B.Arch. DEGREE EXAMINATION, JANUARY 2011

First Semester

381101 – MATHEMATICS

(Common to Interior Design)

(Regulation 2010)

Time: Three hours | Maximum: 100 marks

Answer ALL questions

PART A – (10 X 2 = 20 marks)

1. If $A=\begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}$, then find the eigen values of $2A+3I$.
2. State Cayley-Hamilton theorem.
3. Find the direction ratios of the straight line joining the points $(-2,1,5)$ and $(3,3,2)$.
4. Find the centre and radius of the sphere $x^2+y^2+2x-6y+1=0$.
5. Find the radius of curvature of the curve $x^2+y^2-4x+6y+9=0$.
6. Find the envelope of the curve $y=mx+am^2$, m being a parameter.
7. Find $\frac{du}{dt}$, when $u=x^2+y^2+z^2$, $x=e^t$, $y=e^t\sin t$ and $z=e^t\cos t$.
8. If $x=u(1-v)$ and $y=uv$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
9. Solve $\left(D^2-3D+2\right)y=0$.
10. Find the particular integral of $\left(D^2-2\right)y=e^{2x}$.

PART B – (5 X 16 = 80 marks)

11. (a) (i) If $A=\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$, find A^{-1} using Cayley-Hamilton theorem.

(ii) Find all the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Or

(b) Reduce the quadratic form $x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$ to its canonical form using orthogonal transformation.

12. (a) (i) Find the equation of the plane through the line $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to $x = 2y = 2z$.

(ii) Find the length and equation of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

Or

(b) (i) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z = 8$ is a great circle.

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.

13. (a) (i) Find the radius of curvature at the point $'\theta'$ on the cure $x = 3a\cos\theta - a\cos 3\theta$ and $x = 3a\sin\theta - a\sin 3\theta$.

(ii) Find the envelope of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are parameters connected by $a + b = c$.

Or

(b) (i) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.

(ii) Find the evolute of the parabola $y^2 = 4ax$, considering it as the envelope of its normals.

14. (a) (i) If $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$ and $\phi = f(x, y)$, show that $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}$.

(ii) Expand $e^x \cos y$ at $(0,0)$ upto the third degree.

Or

(b) (i) Examine for the extrema of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.

(ii) Decompose a given number $'a'$ into three positive terms so that their product is a maximum.

15. (a) (i) Solve the equation $\left(D^2-4D+3\right)y=\sin 3x+x^2$.

(ii) Solve the system of equations $\frac{dx}{dt}=3y-2x+5t$ and $\frac{dy}{dt}=3x-2y+2e^{2t}$.

Or

(b) (i) Solve: $x^2y''+4xy'+2y=x^2+\frac{1}{x^2}$.

(ii) Solve $\left(D^2+5D+4\right)y=e^{-x}\sin 2x+2$.