B.Arch. DEGREE EXAMINATION, JANUARY 2014

First Semester

MA2112-MATHEMATICS

(Common to B.Arch. (Interior Design))

(Regulation 2009/2010)

Time: Three hours | Maximum: 100 Marks

Answer ALL questions

PART A – (10 x 2 = 20 marks)

1. Verify that $X = [2, 1, 2]^T$ is an eigenvector corresponding to the eigenvalue $\lambda = 2$ of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.
2. State Cayley-Hamilton theorem.
3. Find the equations of the line passing through $(1, 2, 3)$ and making equal angles with each of the three coordinate axes.
4. Find the equation of the sphere whose centre is $(1, 2, 3)$ and which touches the plane $2x + 2y - z = 2$.
5. Find the radius of curvature of $y^2 = 4ax$ at $y = 2a$.
6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$, m being the parameter.
7. Find $\frac{du}{dt}$ if $u = e^{xy}$ where $x = \sqrt{a^2 - t^2}$ and $y = \sin^3 t$.
8. If $x = u(1+v)$ and $y = v(1+u)$ find the Jacobian of x, y with respect to u, v.
9. Solve $(D^2 - 5)y = 0$.
10. Find the particular integral of $(D^2 + 4D + 5)y = e^{-2x}$.

PART B – (5 x 16 = 80 marks)

11. a) i) Find the eigenvalues and eigenvectors of the following matrix $A=\begin{bmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$.

ii) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}7 & 2 & -2\\ -6 & -1 & 2\\ 6 & 2 & -1\end{bmatrix}$.

(Or)

b) Diagonalise the matrix $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ by orthogonal transformation.

12. a) i) Find the angle between the lines whose direction cosines are given by the equations $3l+m+5n=0$ and $6mn-2nl+5lm=0$.

ii) Show that the lines $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane in which they lie.

(Or)

b) i) Show that the shortest distance between the lines $x+a=2y=-12z$ and $x=y+2a=6z-6a$ is $2a$.

ii) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

13. a) i) Prove that for the curve $y=\frac{ax}{a+x}$, $\left(\frac{2\rho}{a}\right)^{2/3}=\left(\frac{y}{x}\right)^2+\left(\frac{x}{y}\right)^2$, where ρ is the radius of curvature at any point (x,y).

ii) Show that the equation of the evolute of the parabola $x^2=4ay$ is $4(y-2a)^3=24ax^2$.

(Or)

b) i) Show that the line joining any point t on the cycloid $x=a(t+\sin t)$, $y=a(1-\cos t)$ to its centre of curvature is bisected by the line $y=2a$.

ii) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

14. a) i) Find the points on the surface $z^2 = xy+1$ whose distance from the origin is mimimum.

ii) If $u = xyz$, $v = xy + yz + zx$ and $w = x + y + z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(Or)

b) i) Find the dimensions of a rectangular box, without top, of maximum capacity and surface area 432 square meters.

ii) Examine the function $x^3y-3x^2-2y^2-4y-3$ for extreme values.

15. a) i) Solve the equation $\left(D^2+3D+2\right)y=2\sin^2 x+x^2$.

ii) Solve the equation $\left(x^2D^2-xD-3\right)y=\frac{1}{x}\cos(2\log x)$.

(Or)

b) i) Solve the following simultaneous equations $\frac{dx}{dt}+y=\sin t$, $x+\frac{dy}{dt}=\cos t$ given that $x=2$ and $y=0$ at $t=0$.

ii) Solve the equation $\left[\left(3x+2\right)^2 D^2+3\left(3x+2\right)D-36\right]y=3x^2+4x+1$.