SUBJECT NAME : **Probability & Queueing Theory**

SUBJECT CODE : **MA 6453**

MATERIAL NAME : **Part – A questions**

REGULATION : **R2013**

UPDATED ON : **December 2018 (Upto N/D 2018 Q.P)**

TEXTBOOK FOR REFERENCE : **Sri Hariganesh Publications (Author: C. Ganesan)**

To buy the book visit www.hariganesh.com/textbook

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Unit – I (Random Variables)

1) A coin is tossed 2 times, if 'X' denotes the number of heads, find the probability distribution of X.

   Textbook Page No.: 1.80

2) For a binomial distribution with mean 6 and standard deviation $\sqrt{2}$, find the first two terms of the distribution.

   Textbook Page No.: 1.80

3) Given the probability law of Poisson distribution and also its mean and variance.

   Textbook Page No.: 1.82

4) If X and Y are two independent random variables with variances 2 and 3, find the variance of $3X+4Y$.

   Textbook Page No.: 1.82

5) Obtain the mean for a Geometric random variable.

   Textbook Page No.: 1.83

6) If the probability that a target is destroyed on any one shot is 0.5, find the probability that it would be destroyed on 6^{th} attempt.

   Textbook Page No.: 1.84

7) What is meant by memoryless property? Which continuous and discrete distribution follows this property?

   Textbook Page No.: 1.84

8) State memory less property of exponential distribution.

   Textbook Page No.: 1.84

9) Let the random variable X denote the sum obtained in rolling a pair of fair dice. Determine the probability mass function of X.

Textbook Page No.: 1.85

10) A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1+x)$. Find $p(x < 4)$.

Textbook Page No.: 1.85

11) Test whether $f(x) = \begin{cases} |x|, & -1 \le x \le 1 \\ 0, & elsewhere. \end{cases}$ can be the probability density function of a continuous random variable.

Textbook Page No.: 1.86

12) Check whether the following is a probability density function or not:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0,\ \lambda > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Textbook Page No.: 1.87

13) A continuous random variable X has the probability density function given by

$f(x) = \begin{cases} \lambda\left(1+x^2\right), & 1 \le x \le 5 \\ 0, & \text{otherwise} \end{cases}$. Find λ and $P\left(X < 4\right)$.

Textbook Page No.: 1.88

14) A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1+x)$. Find $P\left(X < 4\right)$.

15) A test engineer discovered that the cumulative distribution function of the lifetime of an equipment (in years) is given by $F_X(x) = 1 - e^{\frac{-x}{5}},\ x \ge 0$. What is the expected lifetime of the equipment?

Textbook Page No.: 1.89

16) The cumulative distribution function of the random variable X is given by

$F_X\left(x\right) = \begin{cases} 0, & x < 0 \\ x + \frac{1}{2}, & 0 \le x \le \frac{1}{2} \\ 1, & x > \frac{1}{2} \end{cases}$. Compute $P\left[X > 1/4\right]$.

Textbook Page No.: 1.90

17) If a random variable X has the distribution function $F(X)=\begin{cases}1-e^{-\alpha x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$, where α is the parameter, then find $P(1\le x\le 2)$.

Textbook Page No.: 1.90

18) What do you mean by MGF? Why it is called so?

Textbook Page No.: 1.91

19) If a random variable has the moment generating function given by $M_X(t)=\dfrac{2}{2-t}$, determine the variance of X.

Textbook Page No.: 1.92

20) Every week the average number of wrong-number phone calls received by a certain mail order house is seven. What is the probability that they will receive two wrong calls tomorrow?

21) If X is a normal random variable with mean 3 and variance 9, find the probability that X has between 2 and 5.

Textbook Page No.: 1.93

Unit – II (Two Dimensional Random Variables)

1) The joint pdf of two dimensional random variables (X,Y) is given by

$$f(x,y)=\begin{cases}Kxe^{-y} & ;0<x<2, y>0\\ 0 & ;\text{otherwise}\end{cases}$$. Find the value of K.

Textbook Page No.: 2.64

2) The joint pdf of the RV (X,Y) is given by $f(x,y)=Kxye^{-(x^2+y^2)}, x>0, y>0$. Find the value of K.

Textbook Page No.: 2.64

3) If the joint pdf of (X,Y) is given by $f(x,y)=2$, in $0\le x<y\le 1$, find $E(X)$.

Textbook Page No.: 2.65

4) Let the joint pdf of the random variable (X,Y) be given by

$f(x,y)=4xye^{-(x^2+y^2)};\ x>0$ and $y>0$. Are X and Y independent? Why or why not?

Textbook Page No.: 2.66

5) The joint probability density function of bivariate random variable (X,Y) is given by

$$f(x,y)=\begin{cases} 4xy, & 0<x<1,\ 0<y<1 \\ 0, & otherwise \end{cases}$$. Find $P(X+Y<1)$.

Textbook Page No.: 2.67

6) If X and Y are random variables having the joint density function $f(x,y)=\frac{1}{8}(6-x-y)$,

$0<x<2,\ 2<y<4,$ find $P[x+y<3]$.

Textbook Page No.: 2.68

7) Find the value of k, if the joint density function of (X,Y) is given by

$$f(x,y)=\begin{cases} k(1-x)(1-y), & 0<x<4,\ 1<y<5 \\ 0, & otherwise \end{cases}.$$

Textbook Page No.: 2.69

8) Let the joint probability density function of random variable X and Y be given by

$f(x,y)=8xy,\ 0\le y\le x\le 1$. Calculate the marginal probability density function of X.

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9) Given that joint probability density function of (X,Y) as

$f(x,y)=\frac{1}{6},\ 0<x<2,\ 0<y<3$, determine the marginal density.

Textbook Page No.: 2.71

10) Comment on the statement: "If $Cov(X,Y)=0$, then X and Y are uncorrelated".

Textbook Page No.: 2.72

11) Give a real life example each for positive correlation and negative correlation.

12) Given the two regression lines $3X+12Y=19,\ 3Y+9X=46$, find the coefficient of correlation between X and Y.

13) The regression equations of X on Y and Y on X are respectively $5x-y=22$ and $64x-45y=24$. Find the means of X and Y.

Textbook Page No.: 2.73

14) When will the two regression lines be (a) at right angles (b) coincident?

Textbook Page No.: 2.73

15) If there is no linear correlation between two random variables X and Y, then what can you say about the regression lines?

Textbook Page No.: 2.74

16) Given the RV X with density function $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$. Find the pdf of $Y=8X^3$.

Textbook Page No.: 2.74

Unit – III (Random Processes)

1) Define markov process.

Textbook Page No.: 3.38

2) Define Markov Chain and one-step transition probability.

Textbook Page No.: 3.39

3) When is a Markov chain, called homogeneous?

Textbook Page No.: 3.39

4) Define transition probability matrix.

Textbook Page No.: 3.40

5) Define Strict sense stationary process.

Textbook Page No.: 3.40

6) Define Wide sense stationary process.

Textbook Page No.: 3.40

7) Prove that first order stationary random process has a constant mean.
8) Define Poisson process and state any two properties.

Textbook Page No.: 3.45

9) Examine whether the Poisson process $\{x(t)\}$ is stationary or not.

Textbook Page No.: 3.46

10) Prove that Poisson process is a Markov process.
11) Is a Poisson process a continuous time Markov chain? Justify your answer.
12) If $N(t)$ is the Poisson process, then what can you say about the time we will wait for the first event to occur? And the time we will wait for the nth event to occur?
13) A radioactive source emits particles at a rate of 5 per min in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 min period.

14) A gambler has Rs.2. He bets Rs.1 at a time and wins Rs.1 with probability 1/2. He stops playing if he loses Rs.2 or wins Rs.4. What is the transition probability matrix of the related Markov chain?

15) Consider a random process $x(t)=\cos(\omega t+\theta)$, where ω is a real constant and θ is a uniform variable in $\left(0,\frac{\pi}{2}\right)$. Show that $X(t)$ is not wide sense stationary.

Textbook Page No.: 3.41

16) State Chapman Kolmogorov equations.

Textbook Page No.: 3.42

17) Consider the Markov chain consisting of the three states 0, 1, 2 and transition probability matrix $P=\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & 0\\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4}\\ 0 & \frac{1}{3} & \frac{2}{3}\end{pmatrix}$ it irreducible? Justify.

Textbook Page No.: 3.42

18) Check whether the Markov chain with transition probability matrix $P=\begin{pmatrix}0 & 1 & 0\\ 1/2 & 0 & 1/2\\ 0 & 1 & 0\end{pmatrix}$ is irreducible or not?

Textbook Page No.: 3.43

19) Define Continuous time random process and Discrete state random process.

Textbook Page No.: 3.38

20) Find the transition probability matrix of the process represented by the state transition diagram.

21) If the initial sate probability distribution of a Markov chain is $p^{(0)}=\left(\frac{5}{6}\ \ \frac{1}{6}\right)$ and the transition probability matrix of the chain is $\begin{pmatrix}0 & 1\\ 1/2 & 1/2\end{pmatrix}$, find the probability distribution of the chain after 2 steps.

Textbook Page No.: 3.44

22) If the transition probability matrix (tpm) of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the steady state distribution of the chain.

Textbook Page No.: 3.45

Unit – IV (Queueing Models)

1) Define steady state and transient state in Queuing theory.

Textbook Page No.: 4.52

2) What are the characteristics of a queuing system?

Textbook Page No.: 4.52

3) Define Markovian Queueing models.

Textbook Page No.: 4.52

4) Define $M/M/2$ queueing model. Why the notation M is used?

Textbook Page No.: 4.53

5) State Little's formula for a $(M/M/1):(GD/N/\infty)$ queuing model.

Textbook Page No.: 4.53

6) Draw the state transition diagram for $M/M/1$ queueing model.

Textbook Page No.: 4.54

7) Draw the state transition rate diagram of an $M/M/c$ queueing model.

Textbook Page No.: 4.54

8) State the steady state probabilities of the finite source queueing model represented by $(M/M/R):(GD/K/K)$.

9) What is the steady state condition for $M/M/c$ queueing model?

Textbook Page No.: 4.55

10) What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queueing model represent?

Textbook Page No.: 4.52

11) What is the effective arrival rate for $(M/M/1):(4/FCFS)$ queueing model?

Textbook Page No.: 4.55

12) State the relationship between expected number of customers in the queue and in the system.

Textbook Page No.: 4.56

13) Give a real life situation in which (a) customers are considered for service with last in first out queue discipline (b) a system with infinite number of servers.

14) A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be serviced by the cashier is 24 per hour. Calculate the probability that the cashier is idle.

Textbook Page No.: 4.56

15) Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. What is the average number of customers in the system? and What is the average time of a customer spends in the system?

Textbook Page No.: 4.57

16) Arrival rate of telephone calls at a telephone booth is according to Poisson distribution with an average time of 9 minutes between two consecutive arrivals. The length of a telephone call is assumed to be exponentially distributed with mean 3 minutes. Determine the probability that a person arriving at the both will have to wait.

Textbook Page No.: 4.57

17) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 mins between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 mins. Find the average number of persons waiting in the system.

Textbook Page No.: 4.58

18) Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone calls is assumed to be distributed exponentially with a mean of 3 minutes. What is the average length of the queue that forms from time to time?

Textbook Page No.: 4.58

19) What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queuing system, if $\lambda = 6$ per hour and $\mu = 10$ per hour?

Textbook Page No.: 4.59

20) Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains find the probability that the yard is empty.

Textbook Page No.: 4.59

Unit – V (Advanced Queueing Models)

1) M/G/1 queueing system is Markovian. Comment on this statement.

   Textbook Page No.: 5.33

2) State Pollaczek – Khintchine formula.

   Textbook Page No.: 5.33

3) When a M/G/1 queuing model will become a classic M/M/1 queuing model?

   Textbook Page No.: 5.34

4) State Pollaczek-Khintchine formula for the average number in the system in a $M/G/1$ queueing model and hence derive the same when the service time is constant with mean $\frac{1}{\mu}$.

   Textbook Page No.: 5.33

5) Find the length of the queue for an M/G/1 model if $\lambda = 5$, $\mu = 6$ and $\sigma = \frac{1}{20}$.

   Textbook Page No.: 5.34

6) Define series queue model.

   Textbook Page No.: 5.35

7) Define open network of a queuing system.

   Textbook Page No.: 5.36

8) Define closed network of a queuing system.
9) Distinguish between open and closed networks.
10) Give any two examples for series queuing situations.

    Textbook Page No.: 5.35

11) State Jackson's theorem for an open network.

    Textbook Page No.: 5.37

12) What do you mean by level of multiprogramming in closed queueing network?
13) What do you mean by bottleneck of a network?

    Textbook Page No.: 5.36

14) Consider a tandem queue with 2 independent Markovian servers. The situation at server 1 is just as in an $M/M/1$ model. What will be the type of queue in server 2? Why?

15) Consider a service facility with two sequential stations with respective service rate of 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system, if the system could be approximated by a two stage Tandem queue?

Book for Reference:

"Probability and Queueing Theory"

Edition: 1st Edition

Publication: Sri Hariganesh Publications **Author: C. Ganesan**

Mobile: 9841168917, 8939331876

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----All the Best----