SUBJECT NAME : **Transforms and Partial Differential Equations**

SUBJECT CODE : **MA 6351**

MATERIAL NAME : **Part – A questions**

REGULATION : **R2013**

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TEXT BOOK FOR REFERENCE : **Sri Hariganesh Publications (Author: C. Ganesan)**

To buy the book visit **www.hariganesh.com/textbook**

Unit – I (Partial Differential Equation)

1. Form the PDE from $(x-a)^2+(y-b)^2+z^2=r^2$.

   Text Book Page No.: 1.4

2. Find the PDE of all spheres whose centers lie on the x-axis.

3. Find the PDE of the family of spheres having their centers on the z – axis.

   Text Book Page No.: 1.3

4. Form the partial differential equation by eliminating the constants a and b from $z=(x^2+a^2)(y^2+b^2)$.

   Text Book Page No.: 1.6

5. Form the partial differential equation by eliminating the arbitrary constants a and b from $z=(x^2+a)(y^2+b)$.

   Text Book Page No.: 1.25

6. Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $z=ax^2+by^2$.

   Text Book Page No.: 1.7

7. Form the PDE by eliminating the arbitrary constants a,b from the relation $z=ax^3+by^3$.

   Text Book Page No.: 1.25

8. Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1)=x+ay+b$.

   Text Book Page No.: 1.26

9. Form the partial differential equation by eliminating the arbitrary functions from $f\left(x^2+y^2, z-xy\right)=0$.

10. Find the partial differential equation by eliminating the arbitrary function 'f' from the relation $z=f\left(x^2-y^2\right)$.

11. Eliminate the arbitrary function 'f' from $z=f\left(\frac{y}{x}\right)$ and form the PDE.

    Text Book Page No.: 1.14

12. Form the partial differential equation by eliminating the arbitrary function from $z^2-xy=f\left(\frac{x}{z}\right)$.

    Text Book Page No.: 1.15

13. Form the partial differential equation by eliminating arbitrary function 'f' from $z=e^{ay}f\left(x+by\right)$.

14. Find the partial differential equation of all planes cutting equal intercepts from the x and y axes.

    Text Book Page No.: 1.8

15. Find the complete solution of $p+q=1$. (Text Book Page No.: 1.28)

16. Find the complete integral of $\sqrt{p}+\sqrt{q}=1$.

17. Find the complete solution of the PDE $p^3-q^3=0$.

18. Find the complete integral of $p+q=pq$. (Text Book Page No.: 1.29)

19. Find the complete integral of $\frac{z}{pq}=\frac{x}{q}+\frac{y}{p}+\sqrt{pq}$.

20. Find the complete solution of $q=2px$. (Text Book Page No.: ----)

21. Solve the partial differential equation $pq = x$. (Text Book Page No.: 1.80)

22. Solve the equation $(D - D')^3 z = 0$. (Text Book Page No.: 1.128)

23. Solve $(D^3 - 2D^2D')z = 0$. (Text Book Page No.: 1.127)

24. Solve $(D^2 - 7DD' + D'^2)z = 0$. (Text Book Page No.: 1.123)

25. Solve $(D^3 - D^2D' - 8DD'^2 + 12D'^3)z = 0$.

26. Solve $(D^4 - D'^4)z = 0$. (Text Book Page No.: 1.129)

27. Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-y}$.

    Text Book Page No.: 1.174

28. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$. (Text Book Page No.: 1.179)

29. Solve $(D-1)(D - D' + 1)z = 0$. (Text Book Page No.: 1.180)

Unit – II (Fourier Series)

1. State the Dirichlet's conditions for Fourier series. (Text Book Page No.: 2.1)
2. Write the conditions for a function $f(x)$ to satisfy for the existence of a Fourier series.

   Text Book Page No.: 2.1

3. State the sufficient condition for a function $f(x)$ to be expressed as a Fourier series.

   Text Book Page No.: 2.1

4. Give the expression for the Fourier Series co-efficient b_n for the function

   $f(x) = x \sin x$ defined in $(-2, 2)$.

   Text Book Page No.: 2.77

5. Find the value of a_0 in the Fourier series expansion of $f(x) = e^x$ in $(0, 2\pi)$.

   Text Book Page No.: ----

6. If the Fourier series of the function $f(x)=x,\ -\pi<x<\pi$ with period 2π is given by $f(x)=2\left(\sin x-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\frac{\sin 4x}{4}+\ldots\right)$, then find the sum of the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$.

   Text Book Page No.: 2.77

7. If $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$, deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$.

   Text Book Page No.: ----

8. If $(\pi-x)^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ in $0<x<2\pi$, then deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

   Text Book Page No.: ----

9. Obtain the first term of the Fourier series for the function $f(x)=x^2,\ -\pi<x<\pi$.

   Text Book Page No.: 2.77

10. Find the value of b_n in the Fourier series expansion of $f(x)=\begin{cases}x+\pi & in\ (-\pi,0)\\ -x+\pi & in\ (o,\pi)\end{cases}$.

11. Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi,\pi)$.

    Text Book Page No.: 2.77

12. Expand $f(x)=1$ as a half range sine series in the interval $(0,\pi)$.

    Text Book Page No.: ----

13. Find the half range sine series expansion of $f(x)=1$ in $(0,2)$.

    Text Book Page No.: 2.108

14. Define Root Mean square value of a function $f(x)$ over the interval (a,b).

    Text Book Page No.: 2.93

15. Find the root mean square value of $f(x)=x^2$ in $(0,\ell)$.

Text Book Page No.: 2.95

16. Find the root mean square value of the function $f(x) = x$ in $(0, l)$.

    Text Book Page No.: 2.108

17. Without finding the values of a_0, a_n and b_n, the Fourier coefficients of Fourier series, for the function $F(x) = x^2$ in the interval $(0, \pi)$ find the value of $\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right]$. Text Book Page No.: 2.108

18. Write the complex form of Fourier series for a function $f(x)$ defined in $-\ell < x < \ell$.

19. What is meant by Harmonic Analysis? Text Book Page No.: 2.124

Unit – III (Application of Partial Differential Equations)

1. Classify the partial differential equation $(1-x^2)z_{xx} - 2xyz_{xy} + (1-y^2)z_{yy} + xz_x + 3x^2yz_y - 2z = 0$.

   Text Book Page No.: 3.5

2. Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.

   Text Book Page No.: 3.2

3. Classify the partial differential equation $u_{xx} + u_{xy} = f(x, y)$.

4. Write down all possible solutions of one dimensional wave equation.

   Text Book Page No.: 3.18

5. Write down the three possible solutions of one dimensional heat equation.

   Text Book Page No.: 3.50

6. What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time?

   Text Book Page No.: ----

7. In the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, what does c^2 stand for?

   Text Book Page No.: 3.15

8. In the one dimensional heat equation $u_t = c^2 u_{xx}$, what is c^2?

   Text Book Page No.: 3.46

9. A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. If it is released from rest in this position, write the boundary conditions.

   Text Book Page No.: 3.42

10. Define steady state condition on heat flow.

    Text Book Page No.: 3.55

11. State the assumptions in deriving the one-dimensions heat flow equation.

    Text Book Page No.: ----

12. An insulated rod of length ℓ cm has its ends A and B maintained at 0°C and 80°C respectively. Find the steady state solution of the rod.

    Text Book Page No.: 3.56

13. A rod 40 cm long with insulated sides has its ends A and B kept at 20°C and 60°C respectively. Find the steady state temperature at a location 15 cm from A.

    Text Book Page No.: 3.69

14. A rod 30 cm long has its ends A and B kept at 20°C and 80°C respectively until steady state condition prevail. Find this steady state temperature in the rod.

    Text Book Page No.: 3.69

15. Give three possible solutions of two dimensional steady state heat flow equation.

    Text Book Page No.: 3.78

16. Write all three possible solutions of steady state two – dimensional heat equation.

    Text Book Page No.: 3.78

17. Write down the partial differential equation that represents steady state heat flow in two dimensions and name the variables involved.

    Text Book Page No.: ----

18. Write down the three possible solutions of Laplace equation in two dimensions.

    Text Book Page No.: 3.78

19. Write down the two dimensional heat equation both in transient and steady states.

    Text Book Page No.: ----

20. A plate is bounded by the lines $x = 0, y = 0, x = l$ and $y = l$. Its faces are insulated. The edge coinciding with x – axis is kept at $100^{\circ}C$. The edge coinciding with y – axis is kept at $50^{\circ}C$. The other two edges are kept at $0^{\circ}C$. Write the boundary conditions that are needed for solving two dimensional heat flow equation.

    Text Book Page No.: ----

Unit – IV (Fourier Transform)

1. State Fourier integral theorem. (Text Book Page No.: 4.1)
2. Write the Fourier transform pair. (Text Book Page No.: 4.5)
3. Write the Fourier cosine transform pair. (Text Book Page No.: 4.15)
4. Write the Fourier sine transform pair. (Text Book Page No.: 4.15)
5. Find the Fourier transform of a derivative of the function $f(x)$ if $f(x) \to 0$ as $x \to \pm\infty$.
6. Find the Fourier transform of $e^{-\alpha|x|}$, $\alpha \geq 0$. (Text Book Page No.: 4.18)
7. Find the Fourier cosine transform of e^{-ax}, $x \geq 0$. (Text Book Page No.: 4.46)
8. Find the Fourier sine transform of $f(x) = e^{-ax}$, $a > 0$. (Text Book Page No.: 4.46)
9. Find the Fourier sine transform of e^{-3x}. (Text Book Page No.: 4.67)
10. Find the Fourier sine transform of $\frac{1}{x}$. (Text Book Page No.: 4.57)

11. Find the Fourier transform of $f(x)=\begin{cases} e^{ikx}, & a<x<b \\ 0, & x<a \text{ and } x>b \end{cases}$.

    Text Book Page No.: 4.17

12. Define self-reciprocal with respect to Fourier Transform.

    Text Book Page No.: 4.15

13. State Parseval's identity on Fourier transform. (Text Book Page No.: 4.13)

14. State convolution theorem in Fourier transform. (Text Book Page No.: 4.12)

15. State and prove the change of scale property of Fourier transform.

    Text Book Page No.: 4.7

16. State and prove modulation theorem on Fourier transform.

    Text Book Page No.: 4.8

17. If $F(s)$ is the Fourier transform of $f(x)$, show that $F\left(f(x-a)\right)=e^{ias}F(s)$.

    Text Book Page No.: 4.6

18. What is the Fourier transform of $f(x-a)$, if the Fourier transform of $f(x)$ is $F(s)$?

    Text Book Page No.: 4.6

19. If $F\{f(x)\}=F(s)$, prove that $F\{f(ax)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$.

    Text Book Page No.: 4.7

20. If $F\{f(x)\}=F(s)$, then find $F\left\{e^{iax}f(x)\right\}$. (Text Book Page No.: 4.8)

21. If $F_c(s)$ is the Fourier cosine transform of $f(x)$, prove that the Fourier cosine transform of $f(ax)$ is $\frac{1}{a}F_c\left(\frac{s}{a}\right)$. (Text Book Page No.: 4.43)

Unit – V (Z – Transform)

1. Define the unit step sequence. Write its Z – transform.

   Text Book Page No.: 5.2; 5.9

2. If $Z\{f(n)\} = F(z)$, then show that $Z\{a^n f(n)\} = F\left(\frac{z}{a}\right)$.

   Text Book Page No.: 5.4

3. Find the Z – transform of $\sin\frac{n\pi}{2}$.

   Text Book Page No.: 5.19

4. If $F(z) = \frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)\left(z-\frac{3}{4}\right)}$, find $f(0)$.

   Text Book Page No.: 5.34

5. Find the Z – transform of $x(n) = \begin{cases} \frac{a^n}{n!} & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$.

   Text Book Page No.: 5.17

6. Find the Z – transform of a^n. (Text Book Page No.: 5.12)

7. Find the Z – transform of n. (Text Book Page No.: 5.13)

8. Find the Z – transform of n^2. (Text Book Page No.: 5.14)

9. Find the Z – transform of $\frac{1}{n}$. (Text Book Page No.: 5.14)

10. Find the Z – transform of $\frac{1}{n!}$. (Text Book Page No.: 5.14)

11. Find $Z\{(\cos\theta + i\sin\theta)^n\}$.

12. State initial value theorem on Z – transforms. (Text Book Page No.: 5.32)

13. State the convolution theorem on Z – transforms. (Text Book Page No.: 5.73)

14. Find the inverse Z-transform of $\frac{z}{(z+1)^2}$. (Text Book Page No.: 5.60)

15. Obtain $Z^{-1}\left[\frac{z}{(z+1)(z+2)}\right]$. (Text Book Page No.: 5.59)

16. What advantage is gained when Z – transform is used to solve difference equation?

    Text Book Page No.: ---

17. Form a difference equation by eliminating arbitrary constants from $U_n = A2^{n+1}$.

    Text Book Page No.: 5.111

18. Form a difference equation by eliminating the arbitrary constant A from $y_n = A.3^n$.

    Text Book Page No.: 5.82

19. Form the difference equation by eliminating arbitrary constant 'a' form $y_n = a \cdot 2^n$.

20. Find the difference equation generated by $y_n = an + b2^n$.

    Text Book Page No.: 5.111

21. Solve $y_{n+1} - 2y_n = 0$ given $y_0 = 3$. (Text Book Page No.: 5.111)

Textbook for Reference:

"TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS"

Edition: 3rd Edition

Publication : Hariganesh Publications **Author : C. Ganesan**

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