SUBJECT NAME : Engineering Mathematics – I

SUBJECT CODE : MA2111

MATERIAL NAME : Part – A questions

MATERIAL CODE : JM08AM1013

REGULATION : R2008

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Name of the Student: Branch:

Unit – I (Matrices)

1. Given : $A=\begin{pmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & 2\end{pmatrix}$. Find the eigenvalues of A^2.

2. If λ be an eigenvalue of a non-singular matrix A, show that λ^{-1} is an eigenvalue of A^{-1}.

3. If 3 and 6 are two eigenvalues of $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$, write down all the eigenvalues of A^{-1}.

4. If 1 and 2 are the eigenvalues of a 2 X 2 matrix A, what are the eigenvalues of A^2 and A^{-1}?

5. The product of two eigenvalues of the matrix $A=\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$ is 16. Find the third eigenvalue of A.

6. If the sum of two eigenvalues and trace of a 3 X 3 matrix A are equal, find the value of $|A|$.

7. For a given matrix A of order 3, $|A| = 32$ and two of its eigenvalues are 8 and 2.

8. State Cayley – Hamilton theorem.

9. Use Cayley – Hamilton theorem to find $\left(A^4 - 4A^3 - 5A^2 + A + 2I\right)$ when $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

10. Write down the quadratic form corresponding to the matrix $A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$.

11. Check whether the matrix B is orthogonal? Justify. $B = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

12. Can $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

13. Find the nature of the Quadratic Form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$.

14. Find the symmetric matrix A, whose eigenvalues are 1 and 3 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Unit – II (Three Dimensional Analytical Geometry)

1. Find the centre and radius of the sphere $a\left(x^2 + y^2 + z^2\right) + 2ux + 2vy + 2wz + d = 0$.

2. Find the centre and radius of the sphere $2\left(x^2 + y^2 + z^2\right) + 6x - 6y + 8z + 9 = 0$.

3. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 6x - 6y + 8z + 9 = 0$.

4. Find the equation of the sphere having the points $(2,-3,4)$ and $(-1,5,7)$ as the ends of a diameter.

5. Find the equation of the sphere concentric with $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0$ and passing through the point $(1,2,3)$.

6. Check whether the two spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ are orthogonal.

7. Write the equation of the tangent plane at $(1,5,7)$ to the sphere $(x-2)^2+(y-3)^2+(z-4)^2=14$.

8. Find the equation of the tangent plane at $(-1,4,2)$ on the sphere $x^2+y^2+z^2$ $-2x-4y+2z-3=0$.

9. Find the equation of the tangent plane to the sphere $x^2+y^2+z^2+2x+4y$ $-6z-6=0$ at $(1,2,3)$.

10. Find the equation of the sphere whose centre is $(1,2,3)$ and which touches the plane $2x-y+z+3=0$.

11. Find the equation of the right circular cone whose vertex is at the origin and axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ having semi vertical angel of 45°.

12. Find the equation of the cone whose vertex is the origin and guiding curve is $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{1}=1, x+y+z=1$.

13. Find the equation of the right circular cone whose vertex is the origin, axis is the y – axis, and semi – vertical angle is 30°.

14. Write down the equation of the right circular cone whose vertex is at the origin, semi vertical angel is α and axis is along z-axis.

15. Find the equation of the right circular cylinder whose axis is z-axis and radius is 'a'.

Unit – III (Differential Calculus)

1. For the catenary $y=c\cosh\frac{x}{c}$, find the curvature.

2. Find the radius of curvature for $y=e^x$ at the point where it cuts the y – axis.

3. Define the circle of curvature at a point $p(x_1,y_1)$ on the curve $y=f(x)$.

4. Find the curvature of the curve $2x^2 + 2y^2 + 5x - 2y + 1 = 0$.

5. Find the radius of curvature of the curve $x^2 + y^2 - 4x + 2y - 8 = 0$.

6. Write down the formula for Radius of curvature in terms of Parametric Coordinates System.

7. Find the envelope of the lines $y = mx \pm \sqrt{a^2m^2 + b^2}$ where m is the parameter.

8. Find the envelope of family of straight lines $y = mx + \frac{a}{m}$, m being the parameter.

9. Find the envelope of the family of straight lines $y = mx + \frac{1}{m}$, where m is a parameter.

10. Write the properties of Evolutes.

11. Find the envelope of the family of straight lines $x\cos\theta + y\sin\theta = \alpha$ where θ is the parameter.

12. Find the envelope of the lines $x\operatorname{cosec}\theta - y\cot\theta = a$, θ being the parameter.

13. Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = r^2$, α being the parameter.

14. Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = 4\alpha$, α being the parameter.

Unit – IV (Functions of several variables)

1. Using Euler's theorem, given $u(x, y)$ is a homogeneous function of degree n, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = n(n-1)u$.

2. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

3. Using the definition of total derivative, find the value of $\frac{du}{dt}$ given $u = y^2 - 4ax$; $x = at^2, y = 2at$.

4. If $u = x^3y^2 + x^2y^3$ where $x = at^2$ and $y = 2at$ then find $\frac{du}{dt}$?

5. Find $\frac{du}{dt}$ if $u = \sin(x/y)$, where $x = e^t, y = t^2$.

6. If $u = \frac{y^2}{2x}, v = \frac{x^2+y^2}{2x}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

7. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

8. If $u = 2xy, v = x^2 - y^2, x = r\cos\theta, y = r\sin\theta$ then compute $\frac{\partial(u,v)}{\partial(r,\theta)}$?

9. If $r = \frac{yz}{x}, s = \frac{zx}{y}, t = \frac{xy}{z}$, find $\frac{\partial(r,s,t)}{\partial(x,y,z)}$.

10. Write the sufficient condition for $f(x, y)$ to have a maximum value at (a,b).

11. If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

12. If $u = x^y$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

13. Given $u(x, y) = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$.

14. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

15. If $u = f(y-z, z-x, x-y)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

Unit – V (Multiple Integral)

1. Write down the double integral, to find the area between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

2. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,drd\theta$.

3. Evaluate $\int_0^{2}\int_0^{\pi} r\sin^2\theta\,d\theta dr$.

4. Evaluate $\int_0^{1}\int_x^{\sqrt{x}} xy(x+y)\,dxdy$.

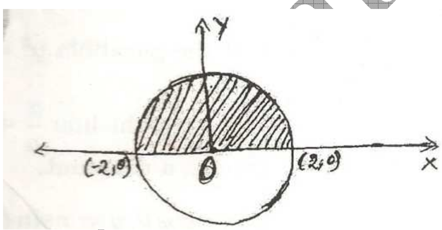
5. Evaluate $\int_0^{1}\int_0^{x^2}\left(x^2+y^2\right)dydx$.

6. Evaluate $\int_0^{a}\int_0^{b}(x+y)\,dxdy$.

7. Evaluate $\int_0^{4}\int_0^{x^2} e^{\frac{y}{x}}\,dydx$.

8. Evaluate $\int_C\left[x^2dy+y^2dx\right]$ where C is the path $y=x$ from $(0,0)$ to $(1,1)$.

9. Evaluate $\iint_R dxdy$, where R is the shaded region in the figure.



10. Change the order of integration in $I=\int_0^{1}\int_{x^2}^{2-x} f(x,y)\,dxdy$.

11. Change the order of integration for the double integral $\int_0^{1}\int_0^{x} f(x,y)\,dxdy$.

12. Change the order of integration in $\int_0^{a}\int_x^{a} f(x,y)\,dydx$.

13. Change the order of integration $\int_0^1 \int_y^1 dxdy$.

14. Express $\int_0^\infty \int_0^\infty f(x,y)\, dxdy$ in polar co-ordinates.

15. Plot the region of integration to evaluate the integral $\iint_D f(x,y)\, dxdy$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

16. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dxdydz$

17. Evaluate $\int_0^1 \int_0^y \int_0^{x+y} dxdydz$.

---- *All the Best* ----