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Name of the Student: **Branch:**

Unit – I (Random Variables)

• Problems on Discrete & Continuous R.Vs

1) A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

a) Find K.

b) Find $P(X<2), P(X>3), P(1<X<5)$.

c) $P(1.5<X<4.5/X>2)$

2) Suppose that X is a continuous random variable whose probability density function is given by $f(x)=\begin{cases} C(4x-2x^2), & 0<x<2 \\ 0, & \text{otherwise} \end{cases}$ (a) find C (b) find $P(X>1)$.

3) A continuous random variable X has the density function $f(x)=\frac{K}{1+x^2}, \ -\infty<x<\infty$. Find the value of K ,the distribution function and $P(X\geq 0)$.

4) A random variable X has the p.d.f $f(x)=\begin{cases} 2x, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$. Find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P\left(\frac{1}{2}<X<\frac{3}{4}\right)$ (iii) $P\left(X>\frac{3}{4}/X>\frac{1}{2}\right)$ (iv) $P\left(X<\frac{3}{4}/X>\frac{1}{2}\right)$.

5) If a random variable X has the p.d.f $f(x)=\begin{cases}\frac{1}{4}, & |x|<2\\ 0, & \text{otherwise}\end{cases}$. Find (a) $P(X<1)$ (b) $P(|X|>1)$ (c) $P(2X+3>5)$

6) The amount of time, in hours that a computer functions before breaking down is a continuous random variable with probability density function given by $f(x)=\begin{cases}\lambda e^{\frac{-x}{100}}, & x\geq 0\\ 0, & x<0\end{cases}$. What is the probability that (a) a computer will function between 50 and 150 hrs. before breaking down (b) it will function less than 500 hrs.

7) A random variable X has the probability density function $f(x)=\begin{cases}\lambda x e^{-x}, & x>0\\ 0, & \text{otherwise}\end{cases}$. Find λ, $c.d.f$, $P(2<X<5)$, $P(X\geq 7)$.

8) A continuous random variable X has the distribution function $F(x)=\begin{cases}0, & x\leq 1\\ k(x-1)^4, & 1<x\leq 3\\ 0, & x>30\end{cases}$. Find k, probability density function $f(x)$, $P(X<2)$.

9) A test engineer discovered that the cumulative distribution function of the lifetime of an equipment in years is given by $F(x)=\begin{cases}1-e^{\frac{-x}{5}}, & x\geq 0\\ 0, & x<0\end{cases}$.
   i) What is the expected life time of the equipment?
   ii) What is the variance of the life time of the equipment?

- **Moments and Moment Generating Function**

1) Find the moment generating function of R.V X whose probability function $P(X=x)=\frac{1}{2^x}, \ x=1,2,\ldots$ Hence find its mean and variance.

2) The density function of random variable X is given by $f(x)=Kx(2-x), \ 0\leq x\leq 2$. Find K, mean, variance and rth moment.

3) Let X be a R.V. with p.d.f $f(x)=\begin{cases}\frac{1}{3}e^{\frac{-x}{3}}, & x>0\\ 0, & \text{Otherwise}\end{cases}$. Find the following
   a) P(X > 3).
   b) Moment generating function of X.
   c) E(X) and Var(X).

4) Find the MGF of a R.V. X having the density function $f(x)=\begin{cases}\frac{x}{2}, & 0\le x\le 2\\ 0, & \text{otherwise}\end{cases}$. Using the generating function find the first four moments about the origin.

• Problems on Distributions

1) The mean of a Binomial distribution is 20 and standard deviation is 4. Determine the parameters of the distribution.
2) If 10% of the screws produced by an automatic machine are defective, find the probability that of 20 screws selected at random, there are (i) exactly two defectives (ii) atmost three defectives (iii) atleast two defectives and (iv) between one and three defectives (inclusive).
3) In a certain factory turning razar blades there is a small chance of 1/500 for any blade to be defective. The blades are in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades respectively in a consignment of 10,000 packets.
4) Prove that the Poisson distribution is a limiting case of binomial distribution.
5) If the mgf of a random variable X is of the form $(0.4e^t+0.6)^8$, what is the mgf of $3X+2$. Evaluate $E(X)$.
6) A discrete R.V. X has moment generating function $M_X(t)=\left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find $E(X)$, $Var(X)$ and $P(X=2)$.
7) If X is a binomially distributed R.V. with $E(X)=2$ and $Var(X)=\frac{4}{3}$, find $P[X=5]$.
8) The number of personal computer (PC) sold daily at a CompuWorld is uniformly distributed with a minimum of 2000 PC and a maximum of 5000 PC. Find the following
    (i) The probability that daily sales will fall between 2,500 PC and 3,000 PC.
    (ii) What is the probability that the CompuWorld will sell at least 4,000 PC's?
    (iii) What is the probability that the CompuWorld will exactly sell 2,500 PC's?
9) Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.8. (i) What is the probability that the target would be hit on 6^{th} attempt? (ii) What is the probability that it takes him less than 5 shots? (iii) What is the probability that it takes him an even number of shots?
10) A die is cast until 6 appears. What is the probability that it must be cast more than 5 times?
11) The length of time (in minutes) that a certain lady speaks on the telephone is found to be random phenomenon, with a probability function specified by the function.

$f(x)=\begin{cases} Ae^{\frac{-x}{5}} & ,x\geq 0 \\ 0, & \text{otherwise} \end{cases}$. (i) Find the value of A that makes f(x) a probability density function. (ii) What is the probability that the number of minutes that she will talk over the phone is (a) more than 10 minutes (b) less than 5 minutes and (c) between 5 and 10 minutes.

12) If the number of kilometers that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 km and if the owner desires to take a 5000 km trip, what is the probability that he will be able to complete his trip without having to replace the car battery? Assume that the car has been used for same time.
13) The mileage which car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tyres will last (i) atleast 20,000 km and (ii) atmost 30,000 km.
14) If X is exponentially distributed with parameter λ, find the value of K there exists $\frac{P(X>k)}{P(X\leq k)}=a$.
15) State and prove memoryless property of Geometric distribution.
16) State and prove memoryless property of Exponential distribution.

Unit – II (Two Dimensional Random Variables)

- **Joint distributions – Marginal & Conditional**

1) The two dimensional random variable (X,Y) has the joint density function $f(x,y)=\frac{x+2y}{27},\ x=0,1,2;\ y=0,1,2$. Find the marginal distribution of X and Y and the conditional distribution of Y given X = x. Also find the conditional distribution of X given Y = 1.
2) The joint probability mass function of (X,Y) is given by $P(x,y)=K(2x+3y),\ x=0,1,2;\ y=1,2,3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $X+Y$ and $P(X+Y>3)$.
3) If the joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases} K(6-x-y) & ,0<x<2,\ 2<y<4 \\ 0 & ,\text{otherwise} \end{cases}$. Find the following (i) the value of K; (ii) $P(x<1,y<3)$; (iii) $P(x+y<3)$; (iv) $P(x<1/y<3)$

4) If the joint pdf of a two – dimensional random variable (X,Y) is given by

$$f(x,y)=\begin{cases} x^2+\dfrac{xy}{3} & ,0<x<1,\ 0<y<2 \\ 0 & ,\text{otherwise}\end{cases}$$. Find (i) $P\left(X>\frac{1}{2}\right)$; (ii) $P(Y<X)$; (iii) $P\left(Y<\frac{1}{2}/X<\frac{1}{2}\right)$. Check whether the conditional density functions are valid.

5) The joint p.d.f of the random variable (X,Y) is given by $f(x,y)=Kxye^{-\left(x^2+y^2\right)}$, $0<x,y<\infty$. Find the value of K and Prove that X and Y are independent.

6) If the joint distribution function of X and Y is given by $F(x,y)=\left(1-e^{-x}\right)\left(1-e^{-y}\right),\quad x>0,y>0$ and "0" otherwise. (i) Are X and Y independent? (ii) Find $P(1<X<3,\ 1<Y<2)$.

• Covariance, Correlation and Regression

1) Define correlation and explain varies type with example.

2) Find the coefficient of correlation between industrial production and export using the following data:

Production (X)	55	56	58	59	60	60	62
Export (Y)	35	38	37	39	44	43	44

3) Let X and Y be discrete random variables with probability function $f(x,y)=\frac{x+y}{21},\ x=1,2,3;\ y=1,2$. Find (i) $Cov(X,Y)$ (ii) Correlation co – efficient.

4) Two random variables X and Y have the following joint probability density function.

$$f(x,y)=\begin{cases} 2-x-y, & 0\le x\le 1,\ 0\le y\le 1 \\ 0, & \text{otherwise}\end{cases}$$. Find $Var(X)$, $Var(Y)$ and the covariance between X and Y. Also find Correlation between X and Y. ($\rho(X,Y)$).

5) Let X and Y be random variables having joint density function.

$$f(x,y)=\begin{cases} \dfrac{3}{2}\left(x^2+y^2\right), & 0\le x,y\le 1 \\ 0, & \text{otherwise}\end{cases}$$. Find the correlation coefficient $\rho(X,Y)$.

6) The independent variables X and Y have the probability density functions given by

$$f_X(x)=\begin{cases} 4ax, & 0\le x\le 1 \\ 0, & \text{otherwise}\end{cases} \qquad f_Y(y)=\begin{cases} 4by, & 0\le y\le 1 \\ 0, & \text{otherwise}\end{cases}$$. Find the correlation coefficient between X and Y.

(or)

The independent variables X and Y have the probability density functions given by

$f_X(x) = \begin{cases} 4ax, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$ $\quad f_Y(y) = \begin{cases} 4by, & 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the correlation coefficient between $X+Y$ and $X-Y$.

7) Let X,Y and Z be uncorrelated random variables with zero means and standard deviations 5, 12 and 9 respectively. If $U = X+Y$ and $V = Y+Z$, find the correlation coefficient between U and V.
8) If the independent random variables X and Y have the variances 36 and 16 respectively, find the correlation coefficient between $X+Y$ and $X-Y$.
9) From the data, find
   (i) The two regression equations.
   (ii) The coefficient of correlation between the marks in Economics and Statistics.
   (iii) The most likely marks in statistics when a mark in Economics is 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

10) The two lines of regression are 8x – 10y + 66 = 0, 40x – 18y – 214 = 0. The variance of X is 9. Find (i) the mean values of X and Y (ii) correlation coefficient between X and Y (iii) Variance of Y.
11) The joint p.d.f of a two dimensional random variable is given by

$f(x,y) = \frac{1}{3}(x+y);\ \ 0 \le x \le 1,\ 0 \le y \le 2$. Find the following

(i) The correlation co – efficient.
(ii) The equation of the two lines of regression
(iii) The two regression curves for mean

- **Transformation of the random variables**

1) If X is a uniformly distributed RV in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, find the pdf of $Y = \tan X$.

2) Let (X,Y) be a two – dimensional non – negative continuous random variables having the joint probability density function $f(x,y)=\begin{cases} 4xye^{-\left(x^2+y^2\right)}, x\geq 0, y\geq 0 \\ 0, \quad \text{elsewhere} \end{cases}$. Find the density function of $U=\sqrt{X^2+Y^2}$.
3) X and Y be independent exponential R.Vs. with parameter 1. Find the j.p.d.f of $U=X+Y$ and $V=\dfrac{X}{X+Y}$.
4) If X and Y are independent exponential random variables each with parameter 1, find the pdf of U = X – Y.
5) Let X and Y be independent random variables both uniformly distributed on (0,1). Calculate the probability density of X + Y.
6) Let X and Y are positive independent random variable with the identical probability density function $f(x)=e^{-x}, x\geq 0$. Find the joint probability density function of $U=X+Y$ and $V=\dfrac{X}{Y}$. Are U and V independent?
7) If the joint probability density of X_1 and X_2 is given by $f(x_1,x_2)=\begin{cases} e^{-(x_1+x_2)}, x_1>0, x_2>0 \\ 0, \quad \text{elsewhere} \end{cases}$, find the probability of $Y=\dfrac{X_1}{X_2+X_2}$.
8) If X is any continuous R.V. having the p.d.f $f(x)=\begin{cases} 2x, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$, and $Y=e^{-X}$, find the p.d.f of the R.V. Y.
9) If the joint p.d.f of the R.Vs X and Y is given by $f(x,y)=\begin{cases} 2, & 0<x<y<1 \\ 0, & \text{otherwise} \end{cases}$ find the p.d.f of the R.V. $U=\dfrac{X}{Y}$.
10) Let X be a continuous random variable with p.d.f $f(x)=\begin{cases} \dfrac{x}{12}, & 1<x<5 \\ 0, & \text{otherwise} \end{cases}$, find the probability density function of 2X – 3.

Unit – III (Random Processes)

• Verification of SSS and WSS process

1) Classify the random process and give example to each.

2) Let $X_n = A\cos(n\lambda) + B\sin(n\lambda)$ where A and B are uncorrelated random variables with $E(A) = E(B) = 0$ and $Var(A) = Var(B) = 1$. Show that X_n is covariance stationary.
3) A stochastic process is described by $X(t) = A\sin t + B\cos t$ where A and B are independent random variables with zero means and equal standard deviations show that the process is stationary of the second order.
4) If $X(t) = Y\cos\omega t + Z\sin\omega t$, where Y and Z are two independent random variables with $E(Y) = E(Z) = 0, E(Y^2) = E(Z^2) = \sigma^2$ and ω is a constants. Prove that $\{X(t)\}$ is a strict sense stationary process of order 2.
5) At the receiver of an AM radio, the received signal contains a cosine carrier signal at the carrier frequency ω_0 with a random phase θ that is uniformly distributed over $(0, 2\pi)$. The received carrier signal is $X(t) = A\cos(\omega_0 t + \theta)$. Show that the process is second order stationary.

- **Problems on Markov Chain**

1) Consider a Markov chain $\{X_n; n \geq 1\}$ with state space $S = \{1, 2\}$ and one – step transition probability matrix $P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$.
   i) Is chain irreducible?
   ii) Find the mean recurrence time of states '1' and '2'.
   iii) Find the invariant probabilities.
2) A raining process is considered as two state Markov chain. If it rains, it is considered to be state 0 and if it does not rain, the chain is in state 1. The transitions probability of the Markov chain is defined as $P = \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$. Find the probability that it will rain for 3 days. Assume the initial probabilities of state 0 and state 1 as 0.4 and 0.6 respectively.
3) A person owning a scooter has the option to switch over to scooter, bike or a car next time with the probability of (0.3, 0.5, 0.2). If the transition probability matrix is $\begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$. What are the probabilities vehicles related to his fourth purchase?
4) Assume that a computer system is in any one of the three states: busy, idle and under repair respectively denoted by 0, 1, 2. Observing its state at 2 pm each day, we get the transition

probability matrix as $P=\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$. Find out the 3rd step transition probability matrix. Determine the limiting probabilities.

- **Poisson process**

1) Queries presented in a computer data base are following a Poisson process of rate $\lambda = 6$ queries per minute. An experiment consists of monitoring the data base for m minutes and recording $N(m)$ the number of queries presented
   i) What is the probability that no queries in a one minute interval?
   ii) What is the probability that exactly 6 queries arriving in one minute interval?
   iii) What is the probability of less than 3 queries arriving in a half minute interval?
2) State the probability law of Poisson process. If $\{X(t)\}$ and $\{Y(t)\}$ are two independent Poisson processes, who that the conditional distribution $\{X(t)\}$ given $\{X(t)+Y(t)\}$ is Binomial.
3) Prove that the Poisson process is Covariance stationary (not a stationary).

Unit – IV (Queueing Models)

- **Model – I (M/M/1) : (∞/FIFO)**

1) Customers arrive at one – man barber shop according to a Poisson process with a mean inter arrival of 12 min. Customers spend an average of 10 min. in the barber's chair.
   a) What is the expected number of customers in the barber shop and in the queue?
   b) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.
   c) How much time can customer expect to spend in the barber's shop?
   d) What is the average time customers spend in the queue?
   e) What is the probability that the waiting time in the system is greater then 30 min.?
   f) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.
   g) What is the probability that more than 3 customers are in the system?
   h) Management will provide another chair and here another barber, when a customer's waiting time in the shop exceeds 1.25 h. How much must the average rate of arrivals increase to warrant a second barber?

2) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min.
    a) Find the average number of persons waiting in the system.
    b) What is the probability that a person arriving at the booth will have to wait in the queue?
    c) What is the probability that it will take him more than 10 min. altogether to wait for the phone and complete his call?
    d) Estimate the fraction of the day when the phone will be in use.
    e) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 min for phon. By how much the flow of arrivals should increase in order to justify a second booth?
3) A duplicating machine maintained for office use is operated by an office assistant who earns Rs.5 per hour. The time to complete each job bares according to an exponential distribution with mean 6 min. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8 hour day is used as a base, determine
    i) the % idle time of the machine
    ii) the average time a job is in the system and
    iii) the average earning per day of the assistant
4) Customers arriving at a watch repair shop according to Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes.
    a) Find the average number of customers L_s in the shop.
    b) Find the average time a customer spends in the shop W_s.
    c) Find the average number of customer in the queue L_q.
    d) What is the probability that the server is idle?
5) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter – arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:
    a) The mean queue size.
    b) The probability that the queue size exceeds 10.
    c) If the input of trains increases to an average of 33 per day what will be change in the above quantities?

- ## Model – II (M/M/s) : (∞/FIFO)

1) A bank has 2 tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time of 3 min. per customer. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate of 16 per hour. Withdrawals also arrive in a Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time for

the customers if each teller could handle both withdrawals and deposits. What could be the effect, if this could only be accomplished by increasing the service time to 3.5 min?

2) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min. and cars arrive for service in a Poisson process at the rate of 30 cars per hour.
    a) What is the probability that an arrival would have to wait in line?
    b) Find the average waiting time, average time spent in the system and the average number of cars in the system.
    c) For what percentage of time would a pump be idle on an average?

- **Model – III (M/M/1) : (K/FIFO)**

1) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.
    a) Find the effective arrival rate at the clinic.
    b) What is the probability that an arriving patient will not wait?
    c) What is the expected waiting time until a patient is discharged from the clinic?
2) In a single server queuing system with Poisson input and exponential service times, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hr and the maximum possible number of calling units in the system is 2, find $P_n \quad (n \geq 0)$, average number of calling units in the system and in the queue and average waiting time in the system and in the queue.
3) A car park contains 5 cars. The arrival of cars is Poisson at a mean rate 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 2 min. How many cars are in the car park on an average and what is the probability of a newly arriving customer finding the car park full and leaving to park his car elsewhere.
4) A one – person barber shop has 6 hairs to accommodate people waiting for a haircut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 min in the barber's chair.
    a) What is the probability that a customer can get directly into the barber's chair upon arrival?
    b) What is the expected number of customers waiting for a haircut?
    c) How much time can a customer expect to spend in the barber shop?
    d) What fraction of potential customers are turned away?

- **Model – IV (M/M/s) : (K/FIFO)**

1) A car servicing station has 2 boys where service can be offered simultaneously. Because of space limitations, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the boys is exponentially distributed with 8 cars

per day. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system.

2) A two person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min. in the barber's chain compute P_0,P_1,P_7,E(N_q) and E(W).
3) A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that each barber service customers according to an exponential distribution with mean of 15 minutes. Further if a customer arrives and there are no empty chairs in the shop, he will leave. What is the probability that the shop is empty? What is the expected number of customers in the shop?

- **Derivations of Queueing Models**

1) Define Kendall's notation. What are the assumptions are made for simplest queuing model.
2) Explain an M/M/1, finite capacity queueing model and obtain expressions for the steady state probabilities for the system size.
3) Explain an M/M/1, finite capacity queueing model and obtain expressions for the steady state probabilities for the system size.
4) Derive the formula for L_s , L_q , W_s and W_q for (M/M/1) : (∞/FIFO).
5) Obtain the expressions for steady state probabilities of a M/M/C queueing system.
6) Explain the model (M/M/S) : (∞/FIFO) (i.e) Multiple server with infinite capacity. Derive the average number of customers in the queue (i.e) average queue length L_{ϕ}.

Unit – V (Non – Markovian Queues and Queue Networks)

- **Pollaczek – Khinchine formula**

1) Consider a single server, Poisson input queue with mean arrival rate of 10 per hour. Currently, the server works according to an exponential distribution with mean service time of 5 minutes. Management has a training course after which service time will following non – exponential distribution, the mean service time will increase to 5.5 min., but the standard deviation will decrease from 5 min. (exponential case) to 4 min. Should the server undergo training?
2) A car manufacturing plant uses one big crane for loading cars into a truck. Cars arrive for loading by the crane according to a Poisson distribution with a mean of 5 cars per hour. Given that the service time for all cars is constant and equal to 6 minutes determine L_S , L_q, W_S and W_q.
3) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars/hr. and may wait in the facility's parking lot if the bay is busy. Find L_S , L_q, W_S and W_q if the service time

a) is constant and equal to 10 minutes.
b) follows uniform distribution between 8 and 12 minutes.
c) follows normal distribution with mean 12 minutes and S.D 3 minutes.
d) follows a discrete distribution with vales 4,8 and 15 minutes with corresponding probabilities 0.2, 0.6 and 0.2.

- **Queueing networks**

Theory Questions

1) Explain how queueing theory could be used to study computer networks.

Series Queues

2) For a 2 – stage (service point) sequential queue model with blockage, compute the average number of customers in the system (L_S) and the average time that a customer has to spend in the system (W_S), if λ= 1, μ_1 = 2 and μ_2 = 1.

Hint: The problem is same as previous, P_{00} = 3/11, P_{11} = 1/11, P_{10} = 2/11, P_{01} = 3/11, P_{b1} = 2/11, L_S = 1, W_S = 11/6.

Open Jackson network

1) In a book shop, there are two sections one for text – books and the other for note – books. Customers from outside arrive at the T.B. section at a Poisson rate of 4 per hour and at the N.B. section at a Poisson rate of 3 per hour. The service rates of the T.B. section and N.B. section are respectively 8 and 10 per hour. A customer upon completion of service at T.B. section is equally likely to go to the N.B. section or to leave the book shop, whereas a customer upon completion of service at N.B. section will go to the T.B. section with probability 1/3 and will leave the book shop otherwise. Find the joint steady – state probability that there are 4 customers in the T.B. section and 2 customers in the N.B. section. Find also the average number of customers in the book shop and the average waiting time of a customer in the shop. Assume that there is only one salesman in each section.

2) In a network of 3 service stations 1, 2, 3 customers arrive at 1, 2, 3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station 1 is equally likely to (i) go to station 2, (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go to station 2 or leave the system.

a) What is the average number of customers in the system, consisting of all the three stations?
b) What is the average time a customer spends in the system?

3) In a departmental store, there are 2 sections namely grocery section and perishable section. Customers from outside arrive at the G – section according to a Poisson process at a mean rate of 10 / hr and they reach the P – section at a mean rate of 2 / hr. The service times at both the sections are exponentially distributed with parameters 15 and 12 respectively. On finishing the job in the G – section, a customer is equally likely to go the P – section or to leave the store, whereas a customer on finishing his job in the P – section will go to the G – section with probability 0.25 and leave the store otherwise. Assuming that there is only one salesman in each sections, find the probability that there are 3 customers in the G – section and 2 customers in the P – section. Find also the average number of customers in the store and the average waiting time of a customer in the store.

Closed Jackson network

1) There are 2 clerks in a union bank one processing housing loan applications and the other processing agricultural loan applications. While processing, they get doubts according to an exponential distribution each with a mean of 1/2. To get clarifications, a clerk goes to the Deputy Manager with probability 3/4 and to the Senior manager with probability 1/4 . After completing the job with D.M., a clerk goes to S.M. with probability 1/3 and returns to his seat otherwise. Completing the job with S.M., a clerk always returns to his seat. If the D.M. clarifies the doubts and advises a clerk according to an exponential distribution with parameter 1 and the S.M. with parameter 3, find

i) The steady – state probabilities $P(n_1,n_2,n_3)$ for all possible values of n_1,n_2,n_3.
ii) The probability that both the managers are idle.
iii) The probability that at least one manager is idle.

---- *All the Best* ----